ESSENTIAL ELEMENT, LINKAGE LEVELS, AND MINI-MAP
MATH: HIGH SCHOOL
M.EE.N-CN.2.A

<table>
<thead>
<tr>
<th>Grade-Level Standard</th>
<th>DLM Essential Element</th>
<th>Linkage Levels</th>
</tr>
</thead>
</table>
| M.N-CN.2.a           | M.EE.N-CN.2.a Use the relation \(i^2 = -1\) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers | Initial Precursor  
- Recognize separateness  
- Recognize set  
- Recognize subset  

Distal Precursor  
- Combine sets  
- Demonstrate the concept of addition  
- Combine  
- Demonstrate the concept of multiplication  
- Solve repeated addition problems  

Proximal Precursor  
- Add 1 and 1  
- Add 1 to 2, 3, and/or 4  
- Add within 5  
- Add within 10  
- Add within 20  
- Multiply by 1, 2, 3, 4, 5, and/or 10  

Target  
- Apply associative property of addition  
- Apply commutative property of addition  
- Apply the commutative property of multiplication  
- Apply the associative property of multiplication  
- Apply the distributive property  

Successor  
- Explain the associative property of addition  
- Explain the commutative property of addition  
- Explain the commutative property of multiplication  
- Explain the distributive property  
- Explain the associative property of multiplication  

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How is the Initial Precursor related to the Target?

**Initial Precursor:** Using the properties of addition and multiplication requires a student to be able to recognize that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numerals to label and count the separate sets. Use tools like the ten-frame to point out whole and parts (e.g., a row of 5 dots and a row of 4 dots are parts or subsets of 9).

![Diagram showing the relationship of nodes in the mini-map]

How is the Distal Precursor related to the Target?

**Distal Precursor:** As students' understanding of labeling and counting sets develops, they will begin working on adding items to a set and combining sets to create a new set. Additionally, students will work on developing an understanding of equal shares by actively participating in one-to-one distribution of objects to person (e.g., giving each person in the group two pencils), objects to objects (e.g., given four counters, students line up four more counters in front of or on top of the first set), and objects to available space (e.g., given three chairs at a table, the student places a cup on the table for each available chair).

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A diagram showing the relationship of nodes in the mini-map appears below.

*Key to map codes in upper right corner of node boxes:*

<table>
<thead>
<tr>
<th>Code</th>
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<tbody>
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<td>DP</td>
<td>Distal Precursor</td>
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<tr>
<td>T</td>
<td>Target</td>
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<tr>
<td>SP</td>
<td>Supporting</td>
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<tr>
<td>S</td>
<td>Successor</td>
</tr>
<tr>
<td>UN</td>
<td>Untested</td>
</tr>
</tbody>
</table>

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M.EE.N-CN.2.a Use the commutative, associative, and distributive properties to add, subtract, and multiply whole numbers.
## Essential Element, Linkage Levels, and Mini-Map
### Math: High School
#### M.EE.N-CN.2.B

<table>
<thead>
<tr>
<th>Grade-Level Standard</th>
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<th>Linkage Levels</th>
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</table>
| M.N-CN.2.b           | M.EE.N-CN.2.b         | Initial Precursor
| Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers | Solve real-world problems involving addition and subtraction of decimals, using models when needed | Recognize set
|                      |                       | Recognize separateness |
|                      |                       | Distal Precursor
|                      |                       | Recognize a unit
|                      |                       | Explain ten as a composition of ten ones |
|                      |                       | Explain place value for ones and tens |
|                      |                       | Proximal Precursor |
|                      |                       | Add 2 decimals with digits in the tenths place |
|                      |                       | Subtract 2 decimals with digits in the tenths place |
|                      |                       | Target |
|                      |                       | Solve word problems involving addition with rational numbers |
|                      |                       | Solve word problems involving subtraction with rational numbers |
|                      |                       | Successor |
|                      |                       | Solve multi-step problems with rational numbers |

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<table>
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<tr>
<th>How is the Initial Precursor related to the Target?</th>
<th>How is the Distal Precursor related to the Target?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Precursor:</strong> Adding and subtracting rational numbers requires a student to be able to recognize that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numerals to label and count the separate sets.</td>
<td><strong>Distal Precursor:</strong> As students’ understanding of numbers develops, they will work with numbers greater than nine (two-digit numbers). Use tools to create tactual and visual models of tens and ones (e.g., ten-frames, connecting cubes, bundling sticks). Educators will describe these numbers as __ groups of ten and __ ones. (e.g., 13 is 1 group of ten and 3 ones).</td>
</tr>
</tbody>
</table>

A diagram showing the relationship of nodes in the mini-map appears below.

*Key to map codes in upper right corner of node boxes:*

- **IP** Initial Precursor
- **SP** Supporting
- **DP** Distal Precursor
- **S** Successor
- **PP** Proximal Precursor
- **UN** Untested
- **T** Target
M.EE.N-CN.2.b Solve real-world problems involving addition and subtraction of decimals, using models when needed.
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</table>
| M.N-CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers | M.EE.N-CN.2.c Solve real-world problems involving multiplication of decimals and whole numbers, using models when needed | Initial Precursor  
• Recognize separateness  
Distal Precursor  
• Recognize a unit  
• Explain place value for ones and tens  
• Explain ten as a composition of ten ones  
Proximal Precursor  
• Multiply 2 decimals with digits in the tenths place  
Target  
• Solve word problems involving multiplication with rational numbers  
Successor  
• Solve multi-step problems with rational numbers |

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<tr>
<td><strong>Initial Precursor:</strong> Solving multiplication problems with or without decimals requires a student to be able to recognize that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numerals to label and count the separate sets.</td>
<td><strong>Distal Precursor:</strong> As students’ understanding of number develops, they will work with numbers greater than nine (two-digit numbers). Use tools to create tactual and visual models of tens and ones (e.g., ten-frames, connecting cubes, bundling sticks). Educators will describe these numbers as _ groups of ten and _ ones. (e.g., 13 is 1 group of ten and 3 ones).</td>
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M.EE.N-CN.2.c Solve real-world problems involving multiplication of decimals and whole numbers, using models when needed.
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| M.N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5 | M.EE.N-RN.1 Determine the value of a quantity that is squared or cubed | Initial Precursor  
- Combine  
- Combine sets  
- Demonstrate the concept of addition  
Distal Precursor  
- Explain repeated addition  
- Represent repeated addition with a model  
- Solve repeated addition problems  
Proximal Precursor  
- Explain product  
- Explain multiplication problems  
- Demonstrate the concept of multiplication  
Target  
- Evaluate expressions with whole number exponents  
Successor  
- Explain perfect cubes  
- Explain perfect squares |
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<td><strong>Initial Precursor:</strong> Determining the value of a quantity that is squared or cubed requires a student to count small amounts, recognizing that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numbers to label and count the separate sets. The general goal is to explore how the set changes when items are combined.</td>
<td><strong>Distal Precursor:</strong> As students gain an understanding of how to group items into sets, educators will begin to help students connect their knowledge of sets with their knowledge of counting and addition. Educators will provide multiple experiences counting sets and combining sets using multiple models. As student understanding progresses, educators provide experience with multiple small sets, and students will use repeated addition to find the total. They can check their work by counting the individual items in each group. Educators should take care to use words like “some,” “all,” “put,” and “add” while defining and demonstrating their meaning. While students do not need to say these words, they do need to learn the meanings.</td>
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M.E.E.N-RN.1 Determine the value of a quantity that is squared or cubed.
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| **M.S-CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”); **M.S-CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent; **M.S-CP.3** Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B; **M.S-CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities; **M.S-CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations | **M.EE.S-CP.1-5** Identify when events are independent or dependent | **Initial Precursor**
- Compare objects for sameness
- Arrange objects in pairs
- Contrast objects
**Distal Precursor**
- Classify
**Proximal Precursor**
- Recognize possible outcomes
- Explain simple events
- Recognize impossible outcomes
**Target**
- Determine if 2 events are independent or dependent
**Successor**
- Explain compound events

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### How is the Initial Precursor related to the Target?

**Initial Precursor:** In order to identify events as independent or dependent (i.e., probability), students begin by learning about attributes, numbers, and measurement. Educators draw student attention to new objects or stimuli, label and describe them (e.g., “this is a circle, so it won’t have any sides,” “this egg carton has 12 spaces, so it is likely that 12 eggs will fit into those spaces,” “this book is a small book, so it’s impossible for it to get bigger”) and students observe, feel, or otherwise interact with the items.

### How is the Distal Precursor related to the Target?

**Distal Precursor:** Proportional understanding is key when working toward describing events as independent or dependent (i.e., probability). Educators provide many opportunities for students to classify (i.e., group) items based on their size (e.g., compare two or more items and determine which is larger or smaller), amount (e.g., numbers larger or smaller than a given number), and distance between numbers (e.g., skip counting by 2, 5, or 10). Educators should also take care to use words like will, won’t, might, likely, unlikely (e.g., “these will go in the same group,” “these won’t go in the same group”) when working with sets. While students do not need to say these words, they do need to learn the meanings.

A diagram showing the relationship of nodes in the mini-map appears below.

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Use a number line or counters to model how you got your answer.

2, 4, 6, ?
M.EE.S-CP.1-5 Identify when events are independent or dependent.
# Essential Element, Linkage Levels, and Mini-Map

**Math: High School**

**M.EE.S-IC.1-2**

<table>
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</table>
| **M.S-IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population; **M.S-IC.2** Decide if a specified model is consistent with results from a given data generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? | **M.EE.S-IC.1-2** Determine the likelihood of an event occurring when the outcomes are equally likely to occur | **Initial Precursor**
- Compare objects for sameness
- Arrange objects in pairs **Distal Precursor**
- Recognize outcomes of an event
- Recognize possible outcomes **Proximal Precursor**
- Recognize sample space **Target**
- Determine theoretical probability of a simple event where all outcomes are equally likely **Successor**
- Determine theoretical probability of simple event where some outcomes are more likely than others |

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<td><strong>Initial Precursor:</strong> In order to determine the likelihood of an event, students begin by learning about attributes, numbers, and measurement. Educators draw student attention to new objects or stimuli, label and describe them (e.g., “this is a circle; it won’t have any sides,” “compare sets of objects, counting them and comparing them using the words same, different, more, less,” “use direct comparison to compare objects”) and students observe, feel, or otherwise interact with the objects.</td>
<td></td>
</tr>
<tr>
<td><strong>Distal Precursor:</strong> Proportional understanding is key when working toward describing events as independent or dependent. Educators provide many opportunities for students to classify (i.e., group) items based on their size (e.g., compare two or more items and determine which is larger or smaller), amount (e.g., numbers larger or smaller than a given number), and distance between numbers (e.g., skip counting by 2, 5, or 10). Educators should also take care to use words like “will,” “won’t,” “might,” “likely,” and “unlikely” when talking about events (e.g., “The traffic lights will change from red to green. The traffic lights won’t change from red to blue,” “A ball is likely to bounce when it is dropped,” “It is unlikely I will travel to the moon”). While students do not need to say these words, they do need to learn the meanings.</td>
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- **DP**  | Distal Precursor  |
- **PP**  | Proximal Precursor|
- **SP**  | Supporting        |
- **S**   | Successor         |
- **UN**  | Untested          |
- **T**   | Target            |
**M.E.E.S-IC.1-2** Determine the likelihood of an event occurring when the outcomes are equally likely to occur.
### ESSENTIAL ELEMENT, LINKAGE LEVELS, AND MINI-MAP

**MATH: HIGH SCHOOL**

**M.EE.G-CO.1**

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</table>
| M.G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc | M.EE.G-CO.1 Know the attributes of perpendicular lines, parallel lines, and line segments, angles, and circles | **Initial Precursor**  
- Recognize same  
- Recognize different  
- Recognize attribute values  
**Distal Precursor**  
- Recognize point  
- Recognize ray  
- Recognize angle  
- Recognize right angles  
**Proximal Precursor**  
- Recognize circles  
- Recognize parallel lines/line segments  
- Recognize perpendicular lines/line segments  
**Target**  
- Define circle  
- Explain angle  
- Explain perpendicular lines/line segments  
- Explain parallel lines/line segments  
**Successor**  
- Explain straight angles  
- Explain adjacent angles  
- Explain vertical angles |
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</table>
| **Initial Precursor:** Knowing the attributes of various shapes, angles, and lines requires a student to first recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., “this is a square”). Then, provide multiple examples of the same shape, so students can make comparisons (e.g., ) focusing student attention on the characteristics that make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as same or different.  

**NOTE:** When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size). | **Distal Precursor:** As students increase their understanding of what makes shapes the same or different, they will begin to learn about other characteristics that make up a shape. The educator will provide multiple objects and tactuals, helping the student explore them and guide the student using hand-under-hand to draw their attention to where lines start and stop (e.g., points and rays) and where two lines meet to make an angle.  

**NOTE:** Recognizing point should only be taught in the context of a lesson on lines, line segments, and angles |

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A diagram showing the relationship of nodes in the mini-map appears below.

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- **SP** Supporting
- **DP** Distal Precursor
- **S** Successor
- **PP** Proximal Precursor
- **UN** Untested
- **T** Target
M.EE.G.CO.1 Know the attributes of perpendicular lines, parallel lines, and line segments, angles, and circles.
### Essential Element, Linkage Levels, and Mini-Map

**Math: High School**

**M.EE.G-CO.4-5**

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</table>
| **M.G-CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments; **M.G-CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another | **M.EE.G-CO.4-5** Given a geometric figure and a rotation, reflection, or translation of that figure, identify the components of the two figures that are congruent | **Initial Precursor**  
- Recognize same  
- Recognize different  
**Distal Precursor**  
- Match the same two-dimensional shapes with same size and different orientations  
- Match the same three-dimensional shapes with same size and different orientation  
**Proximal Precursor**  
- Recognize translation  
- Recognize rotation  
- Recognize reflection  
- Recognize congruent figures  
**Target**  
- Explain the relationship between congruent figures and transformation  
**Successor**  
- Use a sequence of transformations to describe congruence of 2 given figures |

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<tbody>
<tr>
<td><strong>Initial Precursor:</strong> Recognizing congruency requires a student to first recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., &quot;this is a square&quot;). Then, provide multiple examples of the same shape so students can make comparisons, focusing student attention on the characteristics that make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as same or different. <strong>NOTE:</strong> When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size).</td>
<td></td>
</tr>
<tr>
<td><strong>Distal Precursor:</strong> As students develop an understanding of same and different shapes, provide opportunities for students to match or group the same shapes based on the shape size (e.g., &quot;this is a big square,&quot; &quot;this is a little square&quot;). As students progress with identifying the size of shapes, the educator can begin to introduce different orientations of the shape. <strong>NOTE:</strong> As new attributes (e.g., size and orientation) are introduced, be sure to support the student in remembering that the attribute doesn't change the name of the shape.</td>
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- **S** Successor
- **PP** Proximal Precursor
- **UN** Untested
- **T** Target
M.EE.G-CO.4-5 Given a geometric figure and a rotation, reflection, or translation of that figure, identify the components of the two figures that are congruent.
## M.EE.G-CO.6-8

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| **M.G-CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent; **M.G-CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent; **M.G-CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions | **M.EE.G-CO.6-8** Identify corresponding congruent and similar parts of shapes | **Initial Precursor**  
• Recognize same  
• Recognize different  
**Distal Precursor**  
• Match the same two-dimensional shape with different sizes and same orientation  
• Match the same two-dimensional shape with same size and same orientation  
• Match the same three-dimensional shapes with different size and same orientation  
• Match the same three-dimensional shapes with same size and same orientation  
**Proximal Precursor**  
• Recognize congruent figures  
• Recognize similar figures  
**Target**  
• Explain congruent figures  
• Explain similar figures  
**Successor**  
• Explain the relationship between congruent figures and transformation  
• Explain the relationship between similar figures and transformation |

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<table>
<thead>
<tr>
<th>How is the Initial Precursor related to the Target?</th>
<th>How is the Distal Precursor related to the Target?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Precursor:</strong> Recognizing congruent and similar parts of a shape requires a student to first recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., “this is a square”). Then, provide multiple examples of the same shape so students can make comparisons, focusing student attention on the characteristics that make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as same or different.</td>
<td><strong>Distal Precursor:</strong> As students develop an understanding of same and different shapes, provide opportunities for students to match or group the same shapes based on the shape size (e.g., “this is a big square,” “this is a little square”). As students progress with identifying the size of shapes, the educator can begin to introduce different orientations of the shape as well as three-dimensional shapes.</td>
</tr>
</tbody>
</table>

NOTE: When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size).

A diagram showing the relationship of nodes in the mini-map appears below.

Key to map codes in upper right corner of node boxes

<table>
<thead>
<tr>
<th>IP</th>
<th>DP</th>
<th>PP</th>
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</thead>
<tbody>
<tr>
<td>Initial Precursor</td>
<td>Distal Precursor</td>
<td>Proximal Precursor</td>
<td>Target</td>
</tr>
<tr>
<td>SP</td>
<td>S</td>
<td>UN</td>
<td></td>
</tr>
<tr>
<td>Supporting</td>
<td>Successor</td>
<td>Untested</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: As new attributes (e.g., size, orientation, three-dimensional) are introduced, be sure to support the student in remembering that the attribute doesn’t change the name of the shape.
M.EE.G-CO.6-8 Identify corresponding congruent and similar parts of shapes.
<table>
<thead>
<tr>
<th>Grade-Level Standard</th>
<th>DLM Essential Element</th>
<th>Linkage Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.G-MG.1 Use geometric shapes, their measures, and their properties to describe objects; M.G-MG.2 Apply concepts of density based on area and volume in modeling situations; M.G-MG.3 Apply geometric methods to solve design problems</td>
<td>M.EE.G-MG.1-3 Use properties of geometric shapes to describe real-life objects</td>
<td><strong>Initial Precursor</strong>&lt;br&gt;• Recognize same&lt;br&gt;• Recognize different <strong>Distal Precursor</strong>&lt;br&gt;• Match the same two-dimensional shape with same size and same orientation&lt;br&gt;• Match the same two-dimensional shape with different size and same orientation&lt;br&gt;• Match the same three-dimensional shapes with same size and same orientation&lt;br&gt;• Match the same three-dimensional shapes with different size and same orientation <strong>Proximal Precursor</strong>&lt;br&gt;• Recognize squares, circles, triangles, rectangles, cubes, cones, cylinders, and/or spheres <strong>Target</strong>&lt;br&gt;• Use geometric shapes to describe objects <strong>Successor</strong>&lt;br&gt;• Use geometric methods to solve design problems</td>
</tr>
</tbody>
</table>

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## How is the Initial Precursor related to the Target?

**Initial Precursor:** In order to describe real-life objects, students must first recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., “this is a square”). Then provide multiple examples of the same shape so students can make comparisons, focusing student attention on the characteristics that make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as same or different.

**NOTE:** When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size).

## How is the Distal Precursor related to the Target?

**Distal Precursor:** As students develop an understanding of same and different shapes, provide opportunities for students to match or group the same shapes based on the shape size (e.g., “this is a big square,” “this is a little square”). As students progress with identifying the size of shapes, the educator can begin to introduce different orientations of the shape as well as three-dimensional shapes.

**NOTE:** As new attributes (e.g., size, orientation, three-dimensional) are introduced, be sure to support the student in remembering that the attribute doesn’t change the name of the shape.

---

A diagram showing the relationship of nodes in the mini-map appears below.

*Key to map codes in upper right corner of node boxes:*

- **IP** - Initial Precursor
- **SP** - Supporting
- **DP** - Distal Precursor
- **S** - Successor
- **PP** - Proximal Precursor
- **UN** - Untested
- **T** - Target
M.EE.G-MG.1-3 Use properties of geometric shapes to describe real-life objects.
<table>
<thead>
<tr>
<th>Standard</th>
<th>Essential Element</th>
<th>Linkage Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula</td>
<td>M.EE.G-GPE.7 Find perimeters and areas of squares and rectangles to solve real-world problems</td>
<td>Initial Precursor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Recognize attribute values</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distal Precursor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Recognize measureable attributes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Proximal Precursor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Calculate perimeter by adding all the side lengths</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Calculate area by counting unit squares</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Target</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Solve word problems involving perimeter of polygons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Solve word problems involving area of rectangles</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Successor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Mathematize contextual situations involving perimeter of polygons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Mathematize contextual situations involving area of polygons</td>
</tr>
</tbody>
</table>

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</thead>
<tbody>
<tr>
<td><strong>Initial Precursor:</strong> In order to find the perimeter and area of a shape, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “this is a circle, so it does not have sides,” “this is a rectangle, so it has four sides”), and the student observes, feels, or otherwise interacts with the shapes.</td>
<td><strong>Distal Precursor:</strong> As students develop their attention to objects and notice the difference between objects, they will begin working on recognizing and describing measurable attributes. Students need lots of experience making direct comparisons between objects. Educators should use and demonstrate the meaning of comparison words (e.g., big/small, tall/short, longer/shorter). While students do not need to say them, they do need to learn their meaning.</td>
</tr>
</tbody>
</table>

A diagram showing the relationship of nodes in the mini-map appears below.

*Key to map codes in upper right corner of node boxes:*

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</tr>
<tr>
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<td>Target</td>
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</tbody>
</table>
M.EE.G-GPE.7 Find perimeters and areas of squares and rectangles to solve real-world problems.
**ESSENTIAL ELEMENT, LINKAGE LEVELS, AND MINI-MAP**

**MATH: HIGH SCHOOL**

**M.EE.N-Q.1-3**

<table>
<thead>
<tr>
<th>Grade-Level Standard</th>
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</table>
| **M.N-Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays; | **M.EE.N-Q.1-3** Express quantities to the appropriate precision of measurement | **Initial Precursor**  
• Use perceptual subitizing  
**Distal Precursor**  
• Round decimals to any place  
**Proximal Precursor**  
• Solve word problems involving multiplication with rational numbers  
• Solve word problems involving subtraction with rational numbers  
• Solve word problems involving addition with rational numbers  
**Target**  
• Express numerical answers with a degree of precision appropriate for the problem context  
**Successor**  
• Solve multi-step problems with rational numbers |

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### How is the Initial Precursor related to the Target?

**Initial Precursor:** To express quantities with precision, students first need to know number names, the count sequence, one-to-one correspondence, and have cardinality. These procedures and concepts develop through many experiences in early counting. Perceptual subitizing happens when the student is able to name the amount (1–3 items) without actually counting them. For example when an educator asks the student to get their shoes and asks "How many shoes do you have?" the student would reply "two" without using the count sequence of one, two. This only happens when students have been given many experiences counting small numbers with many different contexts and materials.

NOTE: Students who are blind will learn to use tactile enumeration for 1–3 items.

### How is the Distal Precursor related to the Target?

**Distal Precursor:** As students continue to gain experience in counting, educators will introduce the concept that 10 can be grouped into one unit. Educators will use models that help students perceive a group of 10 and some more (e.g., bundles, ten-frames, number line, arrays, etc.). Teen numbers are an important part of understanding this concept. Additionally, educators provide students experience working with money values (e.g., $2.42, $0.67, $5.94) and learning how to round up to the nearest dollar (e.g., $2.42 rounds to $3.00) or tenths place (e.g., $0.67 rounds to $0.70) or ones place (e.g., $5.94 rounds to $5.95). Students should also have experience with rounding down, but not in the context of money (e.g., 0.73 rounds to 70).

A diagram showing the relationship of nodes in the mini-map appears below.

**Key to map codes in upper right corner of node boxes:**

- **IP** Initial Precursor
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- **DP** Distal Precursor
- **S** Successor
- **PP** Proximal Precursor
- **UN** Untested
- **T** Target
M.EE.N-Q.1-3 Express quantities to the appropriate precision of measurement.
## ESSENTIAL ELEMENT, LINKAGE LEVELS, AND MINI-MAP
**MATH: HIGH SCHOOL**

### M.EE.S-ID.1-2

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<tr>
<th>Grade-Level Standard</th>
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</tr>
</thead>
</table>
| **M.S-ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots); **M.S-ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets | **M.EE.S-ID.1-2** Given data, construct a simple graph (line, pie, bar, or picture) and interpret the data | **Initial Precursor**
- Classify
- Order Objects

**Distal Precursor**
- Recognize the structure of a bar graph
- Recognize the structure of a picture graph
- Recognize the structure of a line graph
- Recognize the structure of a pie chart

**Proximal Precursor**
- Use bar graphs to read the data
- Use picture graphs to read the data
- Use line graphs to read the data
- Use pie charts to read the data

**Target**
- Use graphs to read beyond the data
- Represent data using bar graph
- Represent data using picture graph
- Represent data using line graph
- Represent data using pie charts

**Successor**
- Use graphs to read beyond the data

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<tr>
<td><strong>Initial Precursor:</strong> In order to construct a graph, students begin by learning to recognize what is the same and different between familiar items, such as color, shape, quantity, size, texture, and pattern. Educators should take care to use words that describe (e.g., more, less, red circle, same, different) while defining and demonstrating their meaning. While students do not need to say these words, they do need to learn the meanings. Students will also begin to group two or more items in the same set based on an attribute (e.g., two CDs, bumpy balls and bumpy gravel, red rectangles). As the students group two or more items, the educator will demonstrate the representation in graphs and charts and encourage students to actively participate in their creation.</td>
<td><strong>Distal Precursor:</strong> Students actively participate in the creation of bar graphs, picture graphs, line graphs, and pie charts by placing representations for each response to the research question.</td>
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A diagram showing the relationship of nodes in the mini-map appears below.

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M.EE.S-ID.1-2 Given data, construct a simple graph (line, pie, bar, or picture) and interpret the data.
## ESSENTIAL ELEMENT, LINKAGE LEVELS, AND MINI-MAP
### MATH: HIGH SCHOOL
### M.EE.S-ID.3

<table>
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<tr>
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</tr>
</thead>
</table>
| M.S-ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers) | M.EE.S-ID.3 Interpret general trends on a graph or chart | **Initial Precursor**  
- Order objects  
- Classify  
**Distal Precursor**  
- Recognize the structure of a bar graph  
- Recognize the structure of a picture graph  
- Recognize the structure of a line plot (dot plot)  
- Recognize the structure of a pie chart  
**Proximal Precursor**  
- Recognize symmetric distribution  
- Recognize outliers  
- Recognize peaks in data distribution  
- Recognize variability in a date set  
**Target**  
- Analyze overall shape of the data distribution  
- Draw inferences by interpreting general trends on a graph or chart  
**Successor**  
- Draw inferences by comparing two data sets |

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<tr>
<td><strong>Initial Precursor:</strong> In order to construct a graph, students begin by learning to recognize what is the same and different between familiar items, such as color, shape, quantity, size, texture, and pattern. Educators should take care to use words that describe (e.g., more, less, red circle, same, different) while defining and demonstrating their meaning. While students do not need to say these words, they do need to learn the meanings. Students will also begin to group two or more items in the same set based on an attribute (e.g., two CDs, bumpy balls and bumpy gravel, red rectangles). As the students group two or more items, the educator will demonstrate the representation in graphs and charts and encourage students to actively participate in their creation.</td>
<td><strong>Distal Precursor:</strong> Students actively participate in the creation of bar graphs, picture graphs, line graphs, and pie charts by placing representations for each response to the research question.</td>
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M.EE.S-ID.3 Interpret general trends on a graph or chart.
## ESSENTIAL ELEMENT, LINKAGE LEVELS, AND MINI-MAP
### MATH: HIGH SCHOOL

#### M.EE.S-ID.4

<table>
<thead>
<tr>
<th>Grade-Level Standard</th>
<th>DLM Essential Element</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>M.S-ID.4</strong> Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</td>
<td><strong>M.EE.S-ID.4</strong> Calculate the mean of a given data set (limit the number of data points to fewer than five)</td>
<td><strong>Initial Precursor</strong>&lt;br&gt;• Recognize attribute values&lt;br&gt;<strong>Distal Precursor</strong>&lt;br&gt;• Classify&lt;br&gt;<strong>Proximal Precursor</strong>&lt;br&gt;• Summarize data by the number of observations&lt;br&gt;<strong>Target</strong>&lt;br&gt;• Calculate mean&lt;br&gt;<strong>Successor</strong>&lt;br&gt;• Summarize data by measurement</td>
</tr>
</tbody>
</table>

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<tbody>
<tr>
<td><strong>Initial Precursor:</strong> In order to calculate the mean of a data set, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “this is a circle since it does not have any sides,” “two fidgets are big and two fidgets are small”), and the student observes, feels, or otherwise interacts with the shapes. Students also work on counting small units, recognizing that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. As educators present a set, label it (e.g., two balls, one bear, three blocks), count the items, label it again, and encourage students to use numbers to label and count the separate sets.</td>
<td><strong>Distal Precursor:</strong> As students develop their ability to attend to the details of an object and to count objects, educators provide many opportunities for students to classify (group) items based on their size (e.g., compare two or more items and determine which is larger or smaller), amount (e.g., numbers larger or smaller than a given number), and distance between numbers (e.g., skip counting by 2, 5, or 10). Educators should also take care to use attribute words when defining and demonstrating grouping items. While students do not need to say these words, they do need to learn the meanings.</td>
</tr>
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A diagram showing the relationship of nodes in the mini-map appears below.

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- UN Untested
- T Target
M.EE.S-ID.4 Calculate the mean of a given data set (limit the number of data points to fewer than five).
# Essential Element, Linkage Levels, and Mini-Map

## Math: High School

### M.EE.A-CED.1

<table>
<thead>
<tr>
<th>Grade-Level Standard</th>
<th>DLM Essential Element</th>
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</tr>
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</table>
| M.A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions | M.EE.A-CED.1 Create an equation involving one operation with one variable, and use it to solve a real-world problem | **Initial Precursor**  
- Combine sets  
- Partition sets  

**Distal Precursor**  
- Represent multiplication with equations  
- Represent division with equations  
- Represent subtraction with equations  
- Represent addition with equations  

**Proximal Precursor**  
- Represent expressions with variables  
- Represent the unknown in an equation  

**Target**  
- Solve real-world problems using equations with non-negative rational numbers  
- Represent real-world problems as equations  

**Successor**  
- Solve rational equations in 1 variable |

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<td><strong>Initial Precursor:</strong> The knowledge needed to create an equation requires students to manipulate sets (i.e., combining and separating or partitioning). Provide students many opportunities to take a set of objects (e.g., tiles, linking cubes, buttons) and separate them based on a given characteristic (e.g., shape, color, size) into two distinct sets, then separate them again based on another characteristic. Guide students to notice how the set size changes each time you combine or partition the sets.</td>
<td><strong>Distal Precursor:</strong> As students begin to understand labeling and counting sets, they begin to use the number sequence and become more adept at tracking individual objects. Work on this skill using a variety of sets, labeling and counting the sets, and moving items in and out of the sets, labeling and counting the set again. Additionally, the educators will pair those sets with the symbolic representations for addition, subtraction, multiplication, and division (e.g., $3 + 2 = ?$, $3 \times 2 = ?$).</td>
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M.E.E.A-CED.1 Copyright © 2020 University of Kansas Center for Research. All rights reserved.
M.EE.A-CED.1 Create an equations involving one operation with one variable, and use it to solve a real-world problem
# ESSENTIAL ELEMENT, LINKAGE LEVELS, AND MINI-MAP
## MATH: HIGH SCHOOL
### M.EE.A-CED.2-4

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<tr>
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<tbody>
<tr>
<td><strong>M.A-CED.2</strong> Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales;</td>
<td><strong>M.EE.A-CED.2-4</strong> Solve one-step inequalities</td>
<td><strong>Initial Precursor</strong></td>
</tr>
</tbody>
</table>
| **M.A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context; | | • Partition sets  
| **M.A-CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations | | • Combine sets  
| | **Distal Precursor** | • Represent division with equations  
| | | • Represent subtraction with equations  
| | | • Represent addition with equations  
| | | • Represent multiplication with equations  
| | **Proximal Precursor** | • Solve linear equalities in one variable  
| | | • Solve linear inequalities in 1 variable  
| | | • Represent solutions of inequalities on a number line  
| | **Target** | • Explain solution to a linear inequality in one variable  
| | **Successor** | |  

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### How is the Initial Precursor related to the Target?

**Initial Precursor:** The knowledge needed to solve one-step inequalities requires students to manipulate sets (i.e., combining and separating or partitioning). Provide students many opportunities to take a set of objects (e.g., tiles, linking cubes, buttons) and separate them based on a given characteristic (e.g., shape, color, size) into two distinct sets, then separate them again based on another characteristic. Guide students to notice how the set size changes each time you combine or partition the sets.

### How is the Distal Precursor related to the Target?

**Distal Precursor:** As students begin to understand labeling and counting sets, they begin to use the number sequence and become more adept at tracking individual objects. Work on this skill using a variety of sets, labeling and counting the sets, and moving items in and out of the sets, labeling and counting the set again. Additionally, the educators will pair those sets with the symbolic representations for addition, subtraction, multiplication, and division (e.g., $3 + 2 = ?$, $3 \times 2 = ?$).

A diagram showing the relationship of nodes in the mini-map appears below.

**Key to map codes in upper right corner of node boxes:**

- **IP** Initial Precursor
- **SP** Supporting
- **DP** Distal Precursor
- **S** Successor
- **PP** Proximal Precursor
- **UN** Untested
- **T** Target
M.EE.A-CED.2-4 Solve one-step inequalities.
### Grade-Level Standard

**M.A-SSE.1**
Interpret expressions that represent a quantity in terms of its context

### DLM Essential Element

**M.EE.A-SSE.1**
Identify an algebraic expression involving one arithmetic operation to represent a real-world problem

### Linkage Levels

**Initial Precursor**
- Combine sets
- Partition sets

**Distal Precursor**
- Represent subtraction with equations
- Represent addition with equations
- Represent multiplication with equations
- Represent division with equations

**Proximal Precursor**
- Represent the unknown in an equation
- Represent expressions with variables

**Target**
- Represent real-world problems as equations
- Represent real-world problems as expressions

**Successor**
- Solve real-world problems using equations with non-negative rational numbers
<table>
<thead>
<tr>
<th>How is the Initial Precursor related to the Target?</th>
<th>How is the Distal Precursor related to the Target?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Precursor:</strong> The knowledge needed to represent equations requires students to manipulate sets (i.e., combining and separating or partitioning). Provide students many opportunities to take a set of objects (e.g., tiles, linking cubes, buttons) and separate them based on a given characteristic (e.g., shape, color, size) into two distinct sets, then separate them again based on another characteristic. Guide students to notice how the set size changes each time you combine or partition the sets.</td>
<td><strong>Distal Precursor:</strong> As students begin to understand labeling and counting sets, they begin to use the number sequence and become more adept at tracking individual objects. Work on this skill using a variety of sets, labeling and counting the sets, and moving items in and out of the sets, labeling and counting the set again. Additionally, the educators will pair those sets with the symbolic representations for addition, subtraction, multiplication, and division (e.g., $3 + 2 = ?$, $3 \times 2 = ?$).</td>
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- T Target
M.EE.A-SSE.1 Identify an algebraic expression involving one arithmetic operation to represent a real-world problem.
<table>
<thead>
<tr>
<th>Grade-Level Standard</th>
<th>DLM Essential Element</th>
<th>Linkage Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.A-SSE.3</td>
<td>M.EE.A-SSE.3</td>
<td></td>
</tr>
<tr>
<td>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression</td>
<td>Solve simple algebraic equations with one variable using multiplication and division</td>
<td>Initial Precursor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Partition sets</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Combine sets</td>
</tr>
<tr>
<td>Distal Precursor</td>
<td></td>
<td>• Demonstrate the concept of division</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Demonstrate the concept of multiplication</td>
</tr>
<tr>
<td>Proximal Precursor</td>
<td></td>
<td>• Determine the unknown in a division equation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Determine the unknown in a multiplication equation</td>
</tr>
<tr>
<td>Target</td>
<td></td>
<td>• Solve linear equations in one variable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Solve linear equations in 1 variable with rational number coefficients</td>
</tr>
<tr>
<td>Successor</td>
<td></td>
<td>• Solve linear inequalities in 1 variable</td>
</tr>
</tbody>
</table>

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<tr>
<td><strong>Initial Precursor:</strong> The knowledge needed to represent equations requires students to manipulate sets (i.e., combining and separating or partitioning). Provide students many opportunities to take a set of objects (e.g., tiles, linking cubes, buttons) and separate them based on a given characteristic (e.g., shape, color, size) into two distinct sets, then separate them again based on another characteristic. Guide students to notice how the set size changes each time you combine or partition the sets.</td>
<td><strong>Distal Precursor:</strong> As students’ understanding of labeling and counting sets develops, they will begin working on adding items to a set and combining sets to create a new set. Additionally, students will work on developing an understanding of equal shares by actively participating in one-to-one distribution of objects to person (e.g., giving each person in the group two pencils), objects to objects (e.g., given four counters, they would line up four more counters in front of or on top of the first set), and objects to available space (e.g., given three chairs at a table, the student places a cup on the table for each available chair). Students should also experience dividing a whole into equal shares (e.g., having 15 counters and 3 people in the group, give one to each person until there are no more, then count how many each person received).</td>
</tr>
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<td>UN</td>
<td>Successor</td>
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<td>T</td>
<td>Target</td>
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<tr>
<td>SP</td>
<td>Supporting</td>
</tr>
<tr>
<td>UN</td>
<td>Untested</td>
</tr>
</tbody>
</table>

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M.EE.A-SSE.3 Solve simple algebraic equations with one variable using multiplication and division.
## ESSENTIAL ELEMENT, LINKAGE LEVELS, AND MINI-MAP
### MATH: HIGH SCHOOL
#### M.EE.A-REI.10-12

<table>
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<th>Grade-Level Standard</th>
<th>DLM Essential Element</th>
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</tr>
</thead>
</table>
| **M.A-REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line); **M.A-REI.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately; **M.A-REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes | **M.EE.A-REI.10-12** Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas | **Initial Precursor**  
- Arrange objects in pairs  
- Order objects  
**Distal Precursor**  
- Explain coordinate pairs (ordered pairs)  
- Explain x-coordinate  
- Explain y-coordinate  
**Proximal Precursor**  
- Recognize covariation  
- Recognize direction of covariation  
- Describe rate of change in a graph  
**Target**  
- Analyze linear function graphs  
- Interpret a point on the graph of a linear function  
**Successor**  
- Solve real-world problems by interpreting linear function graphs |

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<tbody>
<tr>
<td><strong>Initial Precursor:</strong> In order to analyze function graphs, students begin by learning to notice what is new. The educator draws the students attention to new objects or stimuli, labels them (e.g., “this set has all red objects; this set has all blue,” “these fidgets are big; these fidgets are small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different. Educators provide sorting activities that allow learners to isolate specific attributes while recognizing likenesses and differences among objects. Educators also provide activities that reinforce the skill of ordering (e.g., arrangement of objects from largest to smallest, sequencing daily events, and counting).</td>
<td></td>
</tr>
<tr>
<td><strong>Distal Precursor:</strong> As students’ attention to objects and details develops, educators can extend their attention by providing experience with finding and creating simple patterns using objects and moving to symbols (e.g., numerals). Educators should take care to start with simple patterns (e.g., 1-2-1-2) and take advantage of the symbols that are already being used in the classroom. Educators should demonstrate how students can create and identify the pattern/rule (e.g., using colored cubes, the student creates a line of 5 cubes; the educator then creates a matching set and explains what to do to follow the student’s pattern. Then, the student generates a third matching set. If the order is not followed, it is a good teaching opportunity to talk about why it doesn’t fit the pattern). Learning to identify the rule of patterns will help students extend their thinking across patterns. As students are working on identifying pattern rules, educators can also begin to demonstrate how rules can be used with ordered pairs. Provide students lots of opportunities to apply rules to create their own examples of ordered pairs. Educators should demonstrate how students can use their counting skills to figure out where to mark the point by counting how far along and how far up the x- and y-axes.</td>
<td></td>
</tr>
</tbody>
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A diagram showing the relationship of nodes in the mini-map appears below.

**Key to map codes in upper right corner of node boxes:**

- IP Initial Precursor
- SP Supporting
- DP Distal Precursor
- S Successor
- PP Proximal Precursor
- UN Untested
- T Target
Interpret the meaning of a point on the graph of a line.
# Essential Element, Linkage Levels, and Mini-Map

**Math: High School**

**M.EE.A-SSE.4**

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>M.A-SSE.4</td>
<td>M.EE.A-SSE.4</td>
<td></td>
</tr>
<tr>
<td>Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments</td>
<td>Determine the successive term in a geometric sequence given the common ratio</td>
<td>Initial Precursor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Classify</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Contrast objects</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Order objects</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distal Precursor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recognize symbolic patterns</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recognize sequence</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Proximal Precursor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recognize the recursive rule for geometric sequences</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recognize geometric sequences</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Target</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Extend a geometric sequence by applying the recursive rule</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Successor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Determine the term in a geometric sequence given the nth term formula</td>
</tr>
</tbody>
</table>

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<tbody>
<tr>
<td><strong>Initial Precursor:</strong> In order to determine a geometric sequence (e.g., 2, 4, 8, 16, 32), students begin by learning to notice what is new. The educator draws the students’ attention to new objects or stimuli, labels them (e.g., “there are two cubes,” “this is a circle,” “this fidget is big and this fidget is small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different.</td>
<td><strong>Distal Precursor:</strong> As students develop their understanding of attributes and work toward geometric sequences, educators provide interactive lessons around patterns using attributes like shape, size, and color. At this level, students are also expected to recognize symbolic (e.g., number) patterns. This also requires that students recognize numerals in order (i.e., 1, 2, 3...). Educators should take care to use number names while defining and demonstrating symbolic sequences. While students do not need to say these words, they do need to learn the meanings and the sequence.</td>
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<tr>
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</tbody>
</table>
M.EE.A-SSE.4 Determine the successive term in a geometric sequence given the common ratio.
# Essential Element, Linkage Levels, and Mini-Map

**Math: High School**  
**M.EE.F-BF.1**

<table>
<thead>
<tr>
<th>Grade-Level Standard</th>
<th>DLM Essential Element</th>
<th>Linkage Levels</th>
</tr>
</thead>
</table>
| M.F-BF.1 Write a function that describes a relationship between two quantities | M.EE.F-BF.1 Select the appropriate graphical representation (first quadrant) given a situation involving constant rate of change | **Initial Precursor**  
- Order objects  
- Arrange objects in pairs  

**Distal Precursor**  
- Explain y-coordinate  
- Explain coordinate pairs (ordered pairs)  
- Explain x-coordinate  

**Proximal Precursor**  
- Recognize covariation  
- Recognize direction of covariation  
- Describe rate of change in a graph  

**Target**  
- Represent real-world problems as graphs  

**Successor**  
- Solve real-world problems by interpreting linear function graphs

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<tr>
<td><strong>Initial Precursor:</strong> In order to represent real-world problems on graphs, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “this set has all red objects; this set has all blue,” “these fidgets are big; these fidgets are small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different. Educators provide sorting activities that allow learners to isolate specific attributes while recognizing likenesses and differences among objects. Educators also provide activities that reinforce the skill of ordering (e.g., arrangement of objects from largest to smallest, sequencing daily events, and counting).</td>
<td><strong>Distal Precursor:</strong> As students’ attention to objects and details develops, educators can extend their attention by providing experience with finding and creating simple patterns using objects and moving to symbols (e.g., numerals). Educators should take care to start with simple patterns (e.g., 1-2-1-2) and take advantage of the symbols that are already being used in the classroom. Educators should demonstrate how students can create and identify the pattern/rule (e.g., using colored cubes, the student creates a line of 5 cubes; the educator then creates a matching set and explains what to do to follow the student’s pattern. Then, the student generates a third matching set. If the order is not followed, it is a good teaching opportunity to talk about why it doesn’t fit the pattern). Learning to identify the rule of patterns will help students extend their thinking across patterns. As students are working on identifying pattern rules, educators can also begin to demonstrate how rules can be used with ordered pairs. Provide students lots of opportunities to apply rules to create their own examples of ordered pairs. Educators should demonstrate how students can use their counting skills to figure out where to mark the point by counting how far along and how far up the x- and y-axes.</td>
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<tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
M.EE.F-BF.1 Select the appropriate graphical representation (first quadrant) given a situation involving constant rate of change.
## M.EE.F-BF.2

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</tr>
</thead>
</table>
| M.F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms | M.EE.F-BF.2 Determine an arithmetic sequence with whole numbers when provided a recursive rule | **Initial Precursor**  
- Classify  
- Contrast objects  
- Order objects  

**Distal Precursor**  
- Recognize symbolic patterns  
- Recognize sequence  

**Proximal Precursor**  
- Recognize arithmetic sequences  
- Recognize the recursive rule for arithmetic sequences  

**Target**  
- Extend an arithmetic sequence by applying the recursive rule  

**Successor**  
- Determine the term in an arithmetic sequence given the nth term formula

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<tr>
<td><strong>Initial Precursor:</strong> In order to determine an arithmetic sequence (e.g., 1, 4, 7, 10, 13), students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “there are two cubes,” “this is a circle,” “this fidget is big and this fidget is small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different.</td>
<td><strong>Distal Precursor:</strong> As students develop their understanding of attributes and work towards arithmetic sequences, educators provide interactive lessons around patterns using attributes like shape, size, and color. At this level, students are also expected to recognize symbolic (e.g., numbers) patterns. This also requires that students recognize numerals in order. (i.e., 1, 2, 3...). Educators should take care to use number names while defining and demonstrating symbolic sequences. While students do not need to say these words, they do need to learn the meanings and the sequence.</td>
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- UN: Untested
- T: Target
M.EE.F-BF.2 Determine an arithmetic sequence with whole numbers when provided a recursive rule.
# Essential Element, Linkage Levels, and Mini-Map

## Math: High School

### M.EE.F-IF.1-3

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| **M.F-IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \); **M.F-IF.2** Use function notation, evaluate functions for inputs in their domains, interpret statements that use function notation in terms of a context; **M.F-IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by \( f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1) \) for \( n \geq 1 \) | **M.EE.F-IF.1-3** Use the concept of function to solve problems | **Initial Precursor**  
- Order objects  
- Arrange objects in pairs  
**Distal Precursor**  
- Explain x-coordinate  
- Explain y-coordinate  
- Explain coordinate pairs (ordered pairs)  
**Proximal Precursor**  
- Describe the rate of change in a table  
- Describe rate of change in a graph  
**Target**  
- Solve real-world problems by interpreting linear function graphs  
- Solve real-world problems by interpreting linear function tables  
**Successor**  
- Use graphs to read beyond the data  
- Use tables to predict function values  

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<td><strong>Initial Precursor:</strong> In order to use functions to solve problems, students begin by learning to notice what is new. The educator draws the students’ attention to new objects or stimuli, labels them (e.g., “this set has all red objects; this set has all blue,” “these fidgets are big; these fidgets are small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different. Educators provide sorting activities that allow learners to isolate specific attributes while recognizing likenesses and differences among objects. Educators also provide activities that reinforce the skill of ordering (e.g., arrangement of objects from largest to smallest, sequencing daily events, and counting).</td>
<td><strong>Distal Precursor:</strong> As students’ attention to objects and details develops, educators can extend their attention by providing experience with finding and creating simple patterns using objects and moving to symbols (e.g., numerals). Educators should take care to start with simple patterns (e.g., 1-2-1-2) and take advantage of the symbols that are already being used in the classroom. Educators should demonstrate how students can create and identify the pattern/rule (e.g., using colored cubes, the student creates a line of 5 cubes; the educator then creates a matching set and explains what to do to follow the student’s pattern. Then, the student generates a third matching set. If the order is not followed, it is a good teaching opportunity to talk about why it doesn’t fit the pattern). Learning to identify the rule of patterns will help students extend their thinking across patterns. As students are working on identifying pattern rules, educators can also begin to demonstrate how rules can be used with ordered pairs. Provide students lots of opportunities to apply rules to create their own examples of ordered pairs. Educators should demonstrate how students can use their counting skills to figure out where to mark the point by counting how far along and how far up the x- and y-axes.</td>
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A diagram showing the relationship of nodes in the mini-map appears below.

**Key to map codes in upper right corner of node boxes:**

<table>
<thead>
<tr>
<th>Code</th>
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<tbody>
<tr>
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<td>UN</td>
<td>Untested</td>
</tr>
<tr>
<td>T</td>
<td>Target</td>
</tr>
</tbody>
</table>
M.EE.F-IF.1-3 Use the concept of function to solve problems.
<table>
<thead>
<tr>
<th>Grade-Level Standard</th>
<th>DLM Essential Element</th>
<th>Linkage Levels</th>
</tr>
</thead>
</table>
| **M.F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity; **M.F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function \(h(n)\) gives the number of person-hours it takes to assemble \(n\) engines in a factory, then the positive integers would be an appropriate domain for the function; **M.F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph | **M.EE.F-IF.4-6** Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/ lower, etc. | **Initial Precursor**  
- Arrange objects in pairs  
- Order objects  
**Distal Precursor**  
- Explain coordinate pairs (ordered pairs)  
- Explain x-coordinate  
- Explain y-coordinate  
**Proximal Precursor**  
- Recognize covariation  
- Recognize direction of covariation  
- Describe rate of change in a graph  
**Target**  
- Compare two functions with different rate of change  
- Analyze linear function graphs  
**Successor**  
- Solve real-world problems by interpreting linear function graphs  
- Compare properties of 2 functions represented in the same way |

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### How is the Initial Precursor related to the Target?

**Initial Precursor:** In order to construct graphs that represent a linear function, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “this set has all red objects; this set has all blue,” “these fidgets are big; these fidgets are small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different. Educators provide sorting activities that allow learners to isolate specific attributes while recognizing likenesses and differences among objects. Educators also provide activities that reinforce the skill of ordering (e.g., arrangement of objects from largest to smallest, sequencing daily events, and counting).

### How is the Distal Precursor related to the Target?

**Distal Precursor:** As students’ attention to objects and details develops, educators can extend their attention by providing experience with finding and creating simple patterns using objects and moving to symbols (e.g., numerals). Educators should take care to start with simple patterns (e.g., 1-2-1-2) and take advantage of the symbols that are already being used in the classroom. Educators should demonstrate how students can create and identify the pattern/rule (e.g., using colored cubes, the student creates a line of 5 cubes; the educator then creates a matching set and explains what to do to follow the student’s pattern. Then, the student generates a third matching set. If the order is not followed, it is a good teaching opportunity to talk about why it doesn’t fit the pattern). Learning to identify the rule of patterns will help students extend their thinking across patterns. As students are working on identifying pattern rules, educators can also begin to demonstrate how rules can be used with ordered pairs. Provide students lots of opportunities to apply rules to create their own examples of ordered pairs. Educators should demonstrate how students can use their counting skills to figure out where to mark the point by counting how far along and how far up the x- and y-axes.

A diagram showing the relationship of nodes in the mini-map appears below.

**Key to map codes in upper right corner of node boxes:**

- **IP** Initial Precursor
- **SP** Supporting
- **DP** Distal Precursor
- **S** Successor
- **PP** Proximal Precursor
- **UN** Untested
- **T** Target
M.EE.F-IF.4-6 Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc.
## M.EE.F-LE.1-3

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| M.F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions; **M.F-LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table); **M.F-LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function | M.EE.F-LE.1-3 Model a simple linear function such as \(y=mx\) to show that these functions increase by equal amounts over equal intervals | **Initial Precursor**  
- Arrange objects in pairs  
- Order objects  
**Distal Precursor**  
- Explain x-coordinate  
- Explain y-coordinate  
- Explain coordinate pairs (ordered pairs)  
**Proximal Precursor**  
- Recognize covariation  
- Recognize direction of covariation  
- Determine slope based on coordinate pairs  
**Target**  
- Explain average rate of change  
- Determine rate of change of linear functions  
**Successor**  
- Recognize intervals where function is increasing  
- Recognize intervals where function is decreasing  
- Estimate average rate of change given graph |

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M.EE.F-LE.1-3 Copyright © 2020 University of Kansas Center for Research. All rights reserved. 2 of 3
M.EE.F-LE.1-3 Model a simple linear function such as \( y = mx \) to show that these functions increase by equal amounts over equal intervals.