# Mini-Map for M.EE.HS.N.CN.2.a

Subject: Mathematics
Number and Quantity—The Complex Number System (N.CN)
Grade: 9

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.N.CN.2.a** Use the commutative, associative, and distributive properties to add, subtract, and multiply whole numbers. | **M.N.CN.2.a** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. |

## Linkage Level Descriptions

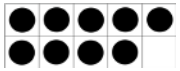
| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Communicate understanding of "separateness" by recognizing objects that are not joined together. Communicate understanding of set by recognizing a group of objects sharing an attribute. Communicate understanding of a subset by recognizing a subset as a set or group of objects within a larger set that share an attribute. | Combine two or more sets to create a new set. Combine two shapes to create a new whole/shape. Solve repeated addition problems by adding the same number multiple times and determining the sum. Demonstrate addition by putting together objects from two sets to create a new set. Demonstrate multiplication by arranging objects into two or more equal groups and communicating that the number of groups times | Multiply numbers up to 12 by factors 1 to 5 and 10, using manipulatives or repeated addition. Add two numbers with a sum within 20 using objects, drawings, counters, or a mathematical equation, and communicate the sum by combining both the numbers. | Apply commutative (e.g., 3 + 4 = 4 + 3) and associative [e.g., (2 + 3) + 5 = (2 + 3) + 5] properties of addition to add two or more numbers. Apply commutative (e.g., 3 × 4 = 4 × 3), associative [e.g., (10 × 4) × 2 = 10 × (4 × 2)], and distributive properties [e.g., 10 × (4 + 2) = (10 × 4) + (10 × 2)] of multiplication as strategies to multiply two or more numbers. | Communicate understanding that the sum of three or more numbers is the same regardless of the grouping or order of addends, the product of three or more numbers is the same regardless of the grouping or order of factors, and multiplying a sum or difference by a given number yields the same result as multiplying each addend by the number and then sum or difference. |

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| | the number of objects in each group equals the total number of objects. | | | |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

Using the properties of addition and multiplication requires a student to be able to recognize that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numerals to label and count the separate sets. Use tools like the ten-frame to point out whole and parts (e.g., a row of 5 dots and a row of 4 dots are parts or subsets of 9).



***How is the Distal Precursor related to the Target?***

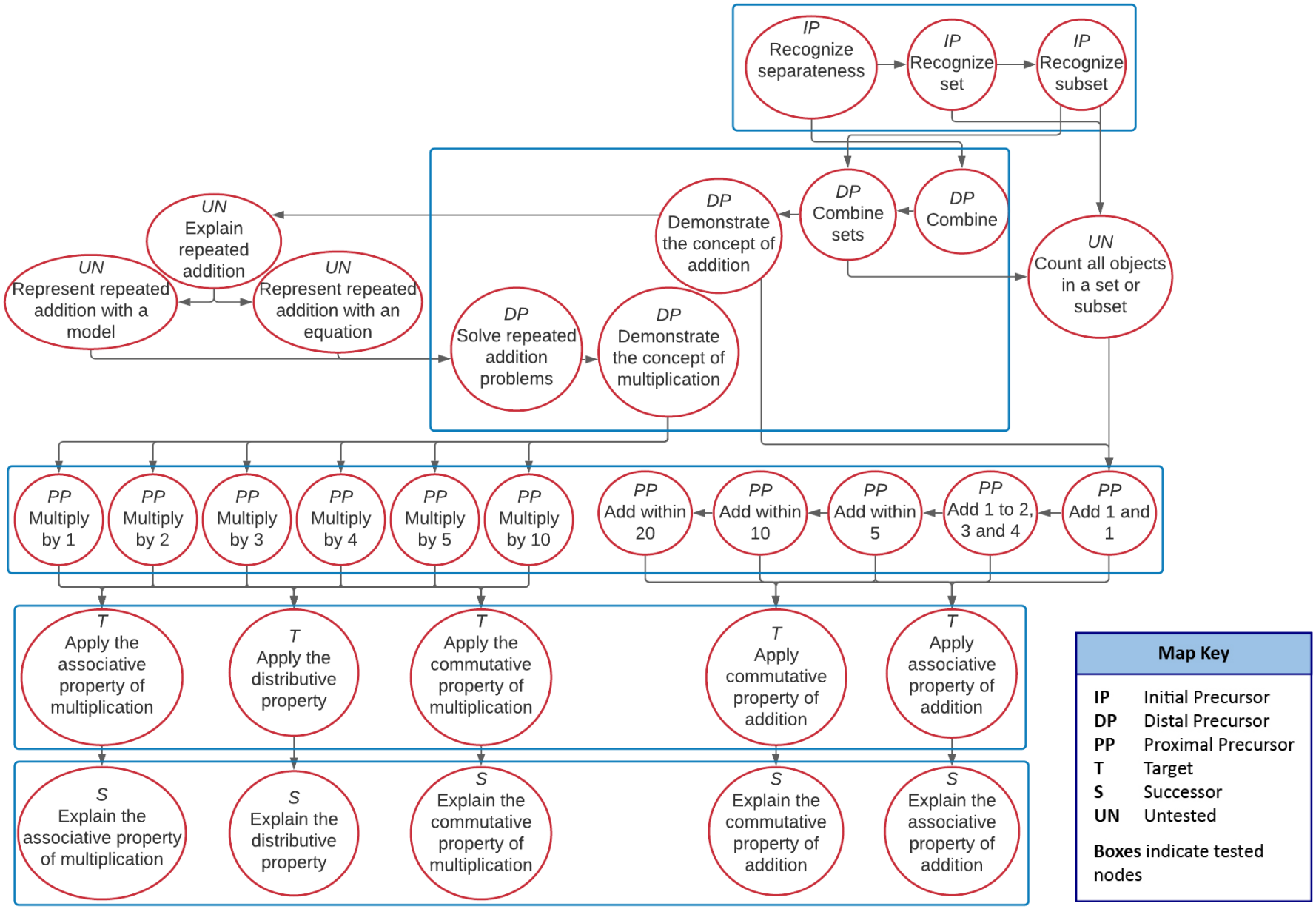
As students' understanding of labeling and counting sets develops, they will begin working on adding items to a set and combining sets to create a new set. Additionally, students will work on developing an understanding of equal shares by actively participating in one-to-one distribution of objects to person (e.g., giving each person in the group two pencils), objects to objects (e.g., given four counters, students line up four more counters in front of or on top of the first set), and objects to available space (e.g., given three chairs at a table, the student places a cup on the table for each available chair).

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.N.CN.2.a** Use the commutative, associative, and distributive properties to add, subtract, and multiply whole numbers.



# Mini-Map for M.EE.HS.N.CN.2.b

Subject: Mathematics
Number and Quantity—The Complex Number System (N.CN)
Grade: 9

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.N.CN.2.b** Solve real-world problems involving addition and subtraction of decimals, using models when needed. | **M.N.CN.2.b** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Communicate understanding of "separateness" by recognizing objects that are not joined together. Communicate understanding of set by recognizing a group of objects sharing an attribute. | Recognize a unit as a group of countable objects. Recognize ten as a group of 10 individual objects or 10 ones. Communicate understanding that the digit in the tens place is formed by grouping objects by 10s and the digit in the ones place is composed of individual objects. | Add and subtract two rational numbers, each with a digit in the tenths places (e.g., subtracting 4.5 from 8.2). | Solve real-world problems involving addition and subtraction of rational numbers with digits to the hundredths place (e.g., John has $2.50. Sara gives him $1.50 more. How much money does John have now?). | Solve multi-step real-world and mathematical problems involving rational numbers with digits to the hundredths place. |

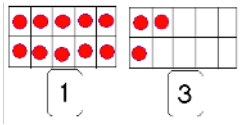
## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

Adding and subtracting rational numbers requires a student to be able to recognize that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numerals to label and count the separate sets.

***How is the Distal Precursor related to the Target?***

As students' understanding of numbers develops, they will work with numbers greater than nine (two-digit numbers). Use tools to create tactual and visual models of tens and ones (e.g., ten-frames, connecting cubes, bundling sticks). Educators will describe these numbers as __ groups of ten and __ ones. (e.g., 13 is 1 group of ten and 3 ones).

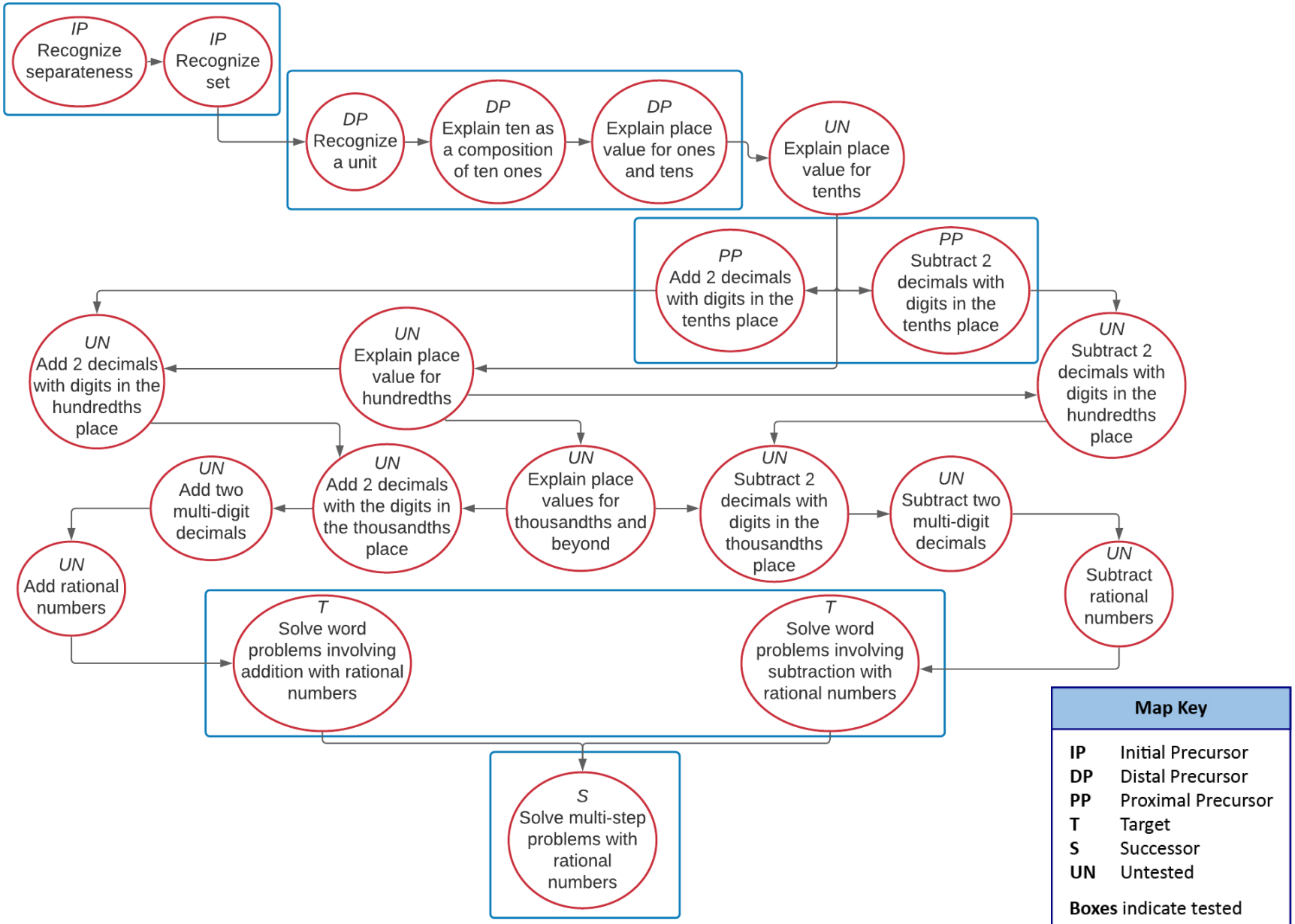



## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.N.CN.2.b** Solve real-world problems involving addition and subtraction of decimals, using models when needed.



# Mini-Map for M.EE.HS.N.CN.2.c

Subject: Mathematics
Number and Quantity—The Complex Number System (N.CN)
Grade: 9

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.N.CN.2.c** Solve real-world problems involving multiplication of decimals and whole numbers, using models when needed. | **M.N.CN.2.c** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Recognize separateness as objects that are not joined together. | Recognize a unit as a group of countable objects. Recognize ten as a group of 10 individual objects or 1 ten. Communicate understanding that the digit in the tens place is formed by grouping objects by 10s and the digit in the ones place is composed of individual objects. | Multiply two rational numbers, each with digits up to the tenth place and limiting the product to answers with tenths, ones, or tens (e.g., multiplying 2.5 by 4.0). | Solve word problems involving multiplication of rational numbers, limiting the factors and products to whole numbers and decimals to the hundredths. | Solve multi-step real-world and mathematical problems involving multiplication of rational numbers, limiting the factors and products to whole numbers and decimals to the hundredths (e.g., Miguel earns $8.75 each day for 5 days. He spends $18.80 on a game. How much money does Miguel have left?). |

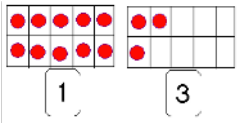
## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

Solving multiplication problems with or without decimals requires a student to be able to recognize that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numerals to label and count the separate sets.

***How is the Distal Precursor related to the Target?***

As students' understanding of number develops, they will work with numbers greater than nine (two-digit numbers). Use tools to create tactual and visual models of tens and ones (e.g., ten-frames, connecting cubes, bundling sticks). Educators will describe these numbers as __ groups of ten and __ ones. (e.g., 13 is 1 group of ten and 3 ones).




## Instructional Resources

| Released Testlets |
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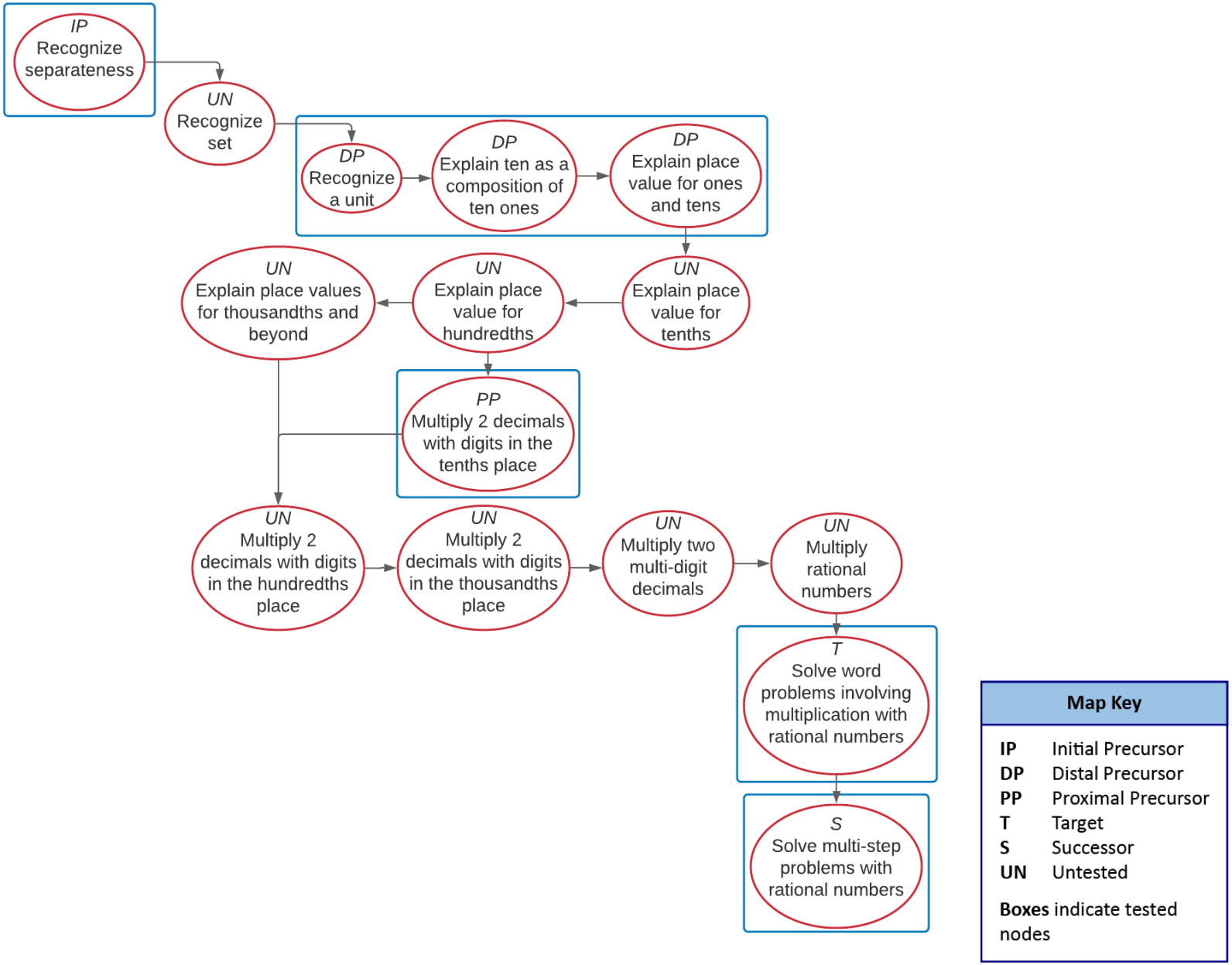
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.N.CN.2.c** Solve real-world problems involving multiplication of decimals and whole numbers, using models when needed.



# Mini-Map for M.EE.HS.N.RN.1

Subject: Mathematics
Number and Quantity—The Real Number System (N.RN)
Grade: 11

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.N.RN.1** Determine the value of a quantity that is squared or cubed. | **M.N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Combine two or more sets of objects to create a new set. Combine two or more parts (e.g., toys, shapes) to form a new whole. Demonstrate an understanding of addition by combining the objects of two or more sets. | Communicate understanding that in repeated addition problems, a single numerical value is added repeatedly (e.g., 6 + 6 + 6) and that one way to add a number a given number of times is by using skip-counting as a strategy (e.g., 6 + 6 + 6 can be added as 6, 12, 18). Represent repeated addition problems using an equation showing the | Demonstrate multiplication by combining multiple sets containing the same number of objects. Communicate understanding that the number of sets times the number of objects in each set equals the total number of objects. Communicate understanding that in multiplication, one factor represents the number of elements in | Determine the value of a whole number exponent expression (e.g., $4^2 = 16$). | Communicate understanding that a perfect square is the product of two equal factors (e.g., 6 x 6) and that a perfect cube is the product of three equal factors (e.g., 7 x 7 x 7). |

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| | addition of the same numeral the required number of times, and find the correct sum using an addition strategy (e.g., 5 + 5 + 5 = 15). | a group, the second factor represents the number of groups, and the product is the number obtained by multiplying two factors. | | |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

Determining the value of a quantity that is squared or cubed requires a student to count small amounts, recognizing that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numbers to label and count the separate sets. The general goal is to explore how the set changes when items are combined.

***How is the Distal Precursor related to the Target?***

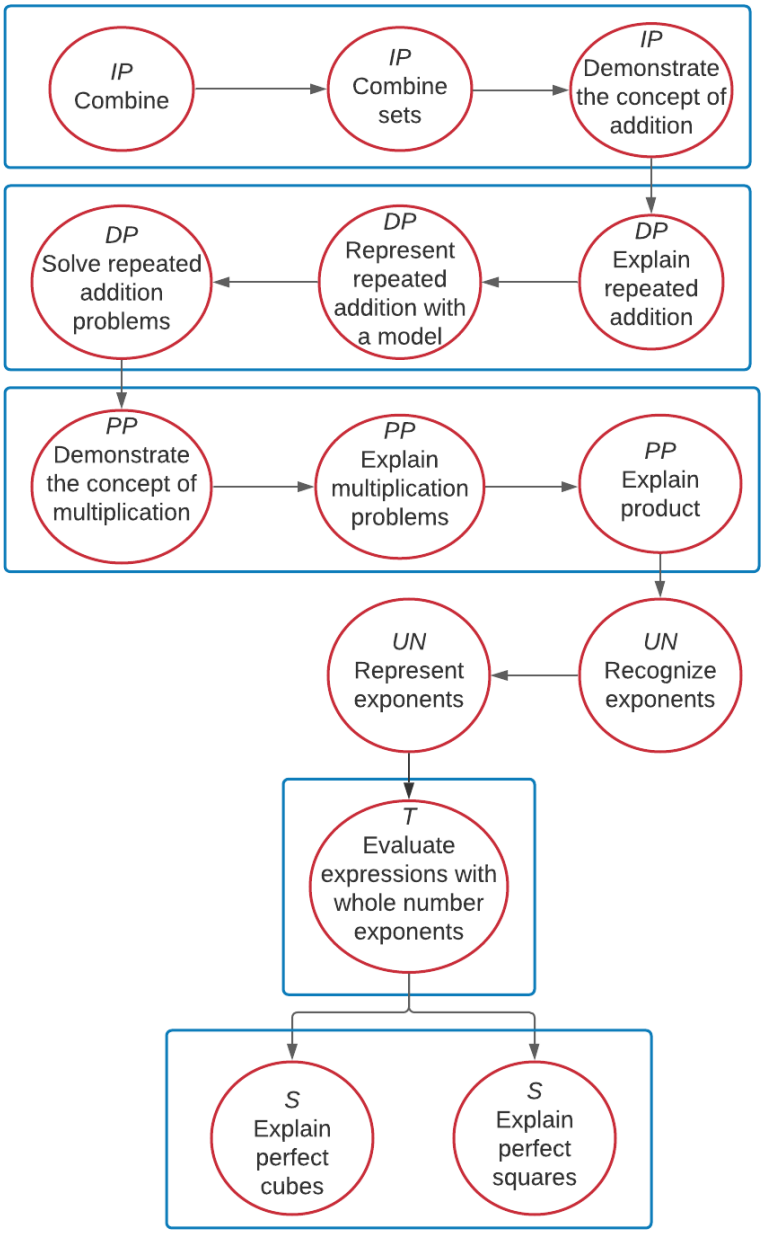
As students gain an understanding of how to group items into sets, educators will begin to help students connect their knowledge of sets with their knowledge of counting and addition. Educators will provide multiple experiences counting sets and combining sets using multiple models. As student understanding progresses, educators provide experience with multiple small sets, and students will use repeated addition to find the total. They can check their work by counting the individual items in each group. Educators should take care to use words like “some,” “all,” “put,” and “add” while defining and demonstrating their meaning. While students do not need to say these words, they do need to learn the meanings.

## Instructional Resources

| **Released Testlets** |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.N.RN.1** Determine the value of a quantity that is squared or cubed.




| Map Key | |
|---|---|
| **IP** | Initial Precursor |
| **DP** | Distal Precursor |
| **PP** | Proximal Precursor |
| **T** | Target |
| **S** | Successor |
| **UN** | Untested |

**Boxes** indicate tested nodes

# Mini-Map for M.EE.HS.S.CP.1-5

Subject: Mathematics
Statistics and Probability—Conditional Probability and the Rules of Probability (S.CP)
Grade: 10

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.S.CP.1-5** Identify when events are independent or dependent. | **M.S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).<br>**M.S.CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.<br>**M.S.CP.3** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.<br>**M.S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.<br>**M.S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Form pairs of objects by matching two objects sharing a specified attribute. Contrast or distinguish objects based on attributes such as shape, size, texture, and numerical pattern. Compare items by attributes such as size, shape, and texture. | Group together objects by attribute values such as shape or size (e.g., group together a square, a rectangle, and a rhombus as they all have four sides). | Recognize possible or impossible outcomes of a simple event. Communicate understanding that a simple event results in a single outcome (e.g., picking a penny from a jar of coins). | Determine if two events are independent or dependent. Communicate understanding that two events are independent if the product of probabilities of two independent events equals the probability of both events occurring together. | Communicate understanding that compound events are comprised of two or more simple events (e.g., getting a heads and an even number when you toss a coin and roll a die). |

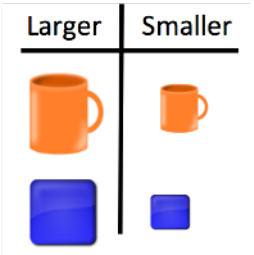
## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

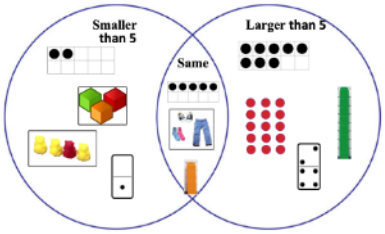
***How is the Initial Precursor related to the Target?***

In order to identify events as independent or dependent (i.e., probability), students begin by learning about attributes, numbers, and measurement. Educators draw student attention to new objects or stimuli, label and describe them (e.g., “this is a circle, so it won't have any sides”, “this egg carton has 12 spaces, so it is likely that 12 eggs will fit into those spaces”, “this book is a small book, so it's impossible for it to get bigger”) and students observe, feel, or otherwise interact with the items.

***How is the Distal Precursor related to the Target?***

Proportional understanding is key when working toward describing events as independent or dependent (i.e., probability). Educators provide many opportunities for students to classify (i.e., group) items based on their size (e.g., compare two or more items and determine which is larger or smaller), amount (e.g., numbers larger or smaller than a given number), and distance between numbers (e.g., skip counting by 2, 5, or 10). Educators should also take care to use words like will, won't, might, likely, unlikely (e.g., “these will go in the same group”, “these won't go in the same group”) when working with sets. While students do not need to say these words, they do need to learn the meanings.



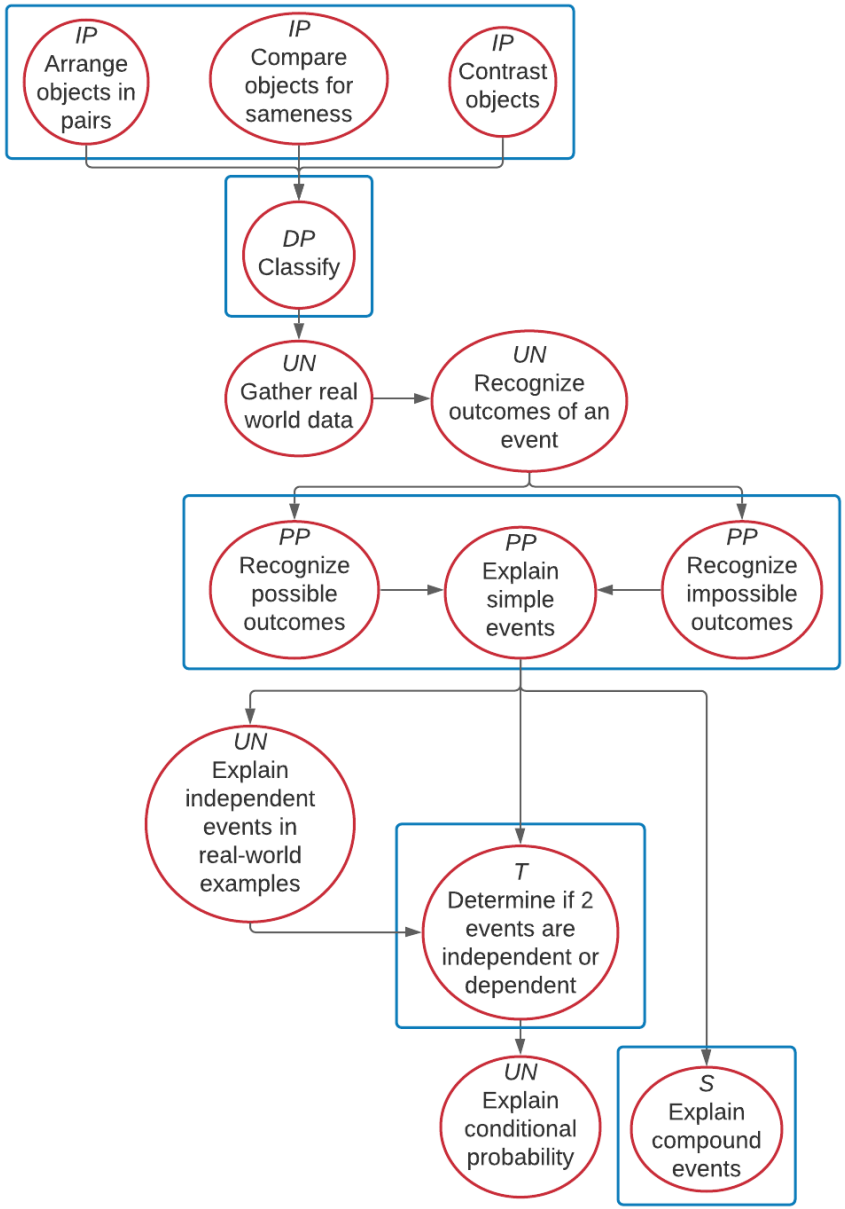



## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.S.CP.1-5** Identify when events are independent or dependent.




| Map Key | |
|---|---|
| **IP** | Initial Precursor |
| **DP** | Distal Precursor |
| **PP** | Proximal Precursor |
| **T** | Target |
| **S** | Successor |
| **UN** | Untested |

**Boxes** indicate tested nodes

# Mini-Map for M.EE.HS.S.IC.1-2

Subject: Mathematics
Statistics and Probability—Making Inferences and Justifying Conclusions (S.IC)
Grade: 11

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.S.IC.1-2** Determine the likelihood of an event occurring when the outcomes are equally likely to occur. | **M.S.IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.<br>**M.S.IC.2** Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation). For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Compare the attributes of two objects to identify common characteristics. Create a pair by joining two separate objects. | Recognize outcomes of an event that are either possible or impossible (e.g., shown a picture of a girl standing in the rain with no umbrella, the student identifies possible outcomes such as wet hair or wet clothes). | Recognize an event's sample space by identifying all the possible outcomes of an event (e.g., identify all possible outcomes of rolling a six-sided number cube as numbers 1-6). | Determine the probability of simple events where all outcomes are equally likely (e.g., the theoretical probability of getting a 4 when rolling a six-sided number cube is 1/6). | Determine the theoretical probability of a simple event where some outcomes are more likely than others (e.g., drawing a green marble out of a bag where there are 2 blue marbles, 7 green marbles, and 3 red marbles is 7/12). |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to determine the likelihood of an event, students begin by learning about attributes, numbers, and measurement. Educators draw student attention to new objects or stimuli, label and describe them (e.g., “this is a circle; it won't have any sides”, “compare sets of objects, counting them and comparing them using the words same, different, more, less”, “use direct comparison to compare objects”) and students observe, feel, or otherwise interact with the objects.

***How is the Distal Precursor related to the Target?***

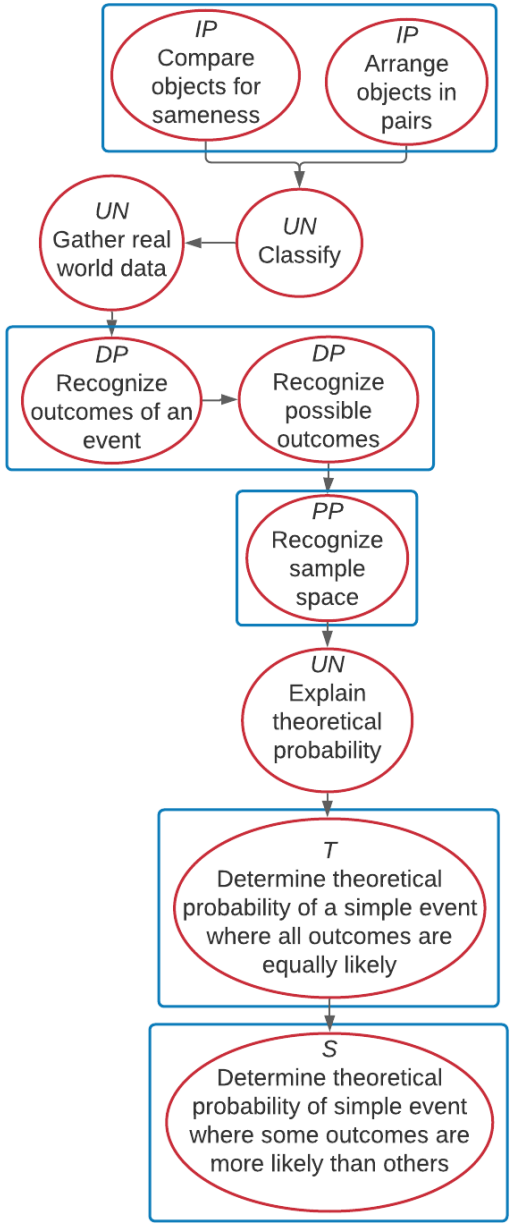
Proportional understanding is key when working toward describing events as independent or dependent. Educators provide many opportunities for students to classify (i.e., group) items based on their size (e.g., compare two or more items and determine which is larger or smaller), amount (e.g., numbers larger or smaller than a given number), and distance between numbers (e.g., skip counting by 2, 5, or 10). Educators should also take care to use words like “will”, “won't”, “might”, “likely”, and “unlikely” when talking about events (e.g., “The traffic lights will change from red to green. The traffic lights won’t change from red to blue.”, “A ball is likely to bounce when it is dropped.”, “It is unlikely I will travel to the moon.”). While students do not need to say these words, they do need to learn the meanings.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.S.IC.1-2** Determine the likelihood of an event occurring when the outcomes are equally likely to occur.




| Map Key | |
|---|---|
| **IP** | Initial Precursor |
| **DP** | Distal Precursor |
| **PP** | Proximal Precursor |
| **T** | Target |
| **S** | Successor |
| **UN** | Untested |

**Boxes** indicate tested nodes

# Mini-Map for M.EE.HS.G.CO.1

Subject: Mathematics
Geometry—Congruence (G.CO)
Grade: 9

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.G.CO.1** Know the attributes of perpendicular lines, parallel lines, and line segments; angles; and circles. | **M.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Recognize "same" as the object that shares all of the same attributes as other objects in a group.<br>Recognize "different" as the object that shares some or none of the attributes as other objects in a group.<br>Recognize attributes or characteristics of an object, such as color, orientation, length, width, and weight. | Identify the symbols that represent a point, ray, angle, and right angle. | Recognize the shape that represents a circle.<br>Recognize lines or line segments that intersect at a 90-degree angle as perpendicular lines or line segments.<br>Recognize lines or line segments that are equal distance apart and do not intersect at any point as parallel lines or line segments. | Communicate understanding that perpendicular lines intersect at a 90-degree angle and parallel lines are equal distance apart and do not intersect at any point.<br>Communicate understanding that an angle is a figure (or shape) formed by two rays meeting at a common endpoint, and a circle is a two-dimensional shape that has an outline or circumference that only contains points that are | Communicate understanding that vertical angles are angles that are equal in measure and share a vertex, a straight angle is an angle that has a measurement of 180 degrees, and adjacent angles are angles next to each other that share a ray and a vertex. |

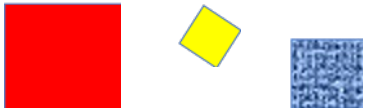
| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| | | | equidistant from a common point, called the center. | |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

Knowing the attributes of various shapes, angles, and lines requires a student to first recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., "this is a square" ). Then, provide multiple examples of the same shape, so students can make comparisons (e.g., ) focusing student attention on the characteristics that make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as same or different.

NOTE: When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size).



***How is the Distal Precursor related to the Target?***

As students increase their understanding of what makes shapes the same or different, they will begin to learn about other characteristics that make up a shape. The educator will provide multiple objects and tactuals, helping the student explore them and guide the student using hand-under-hand to draw their attention to where lines start and stop (e.g., points and rays) and where two lines meet to make an angle.

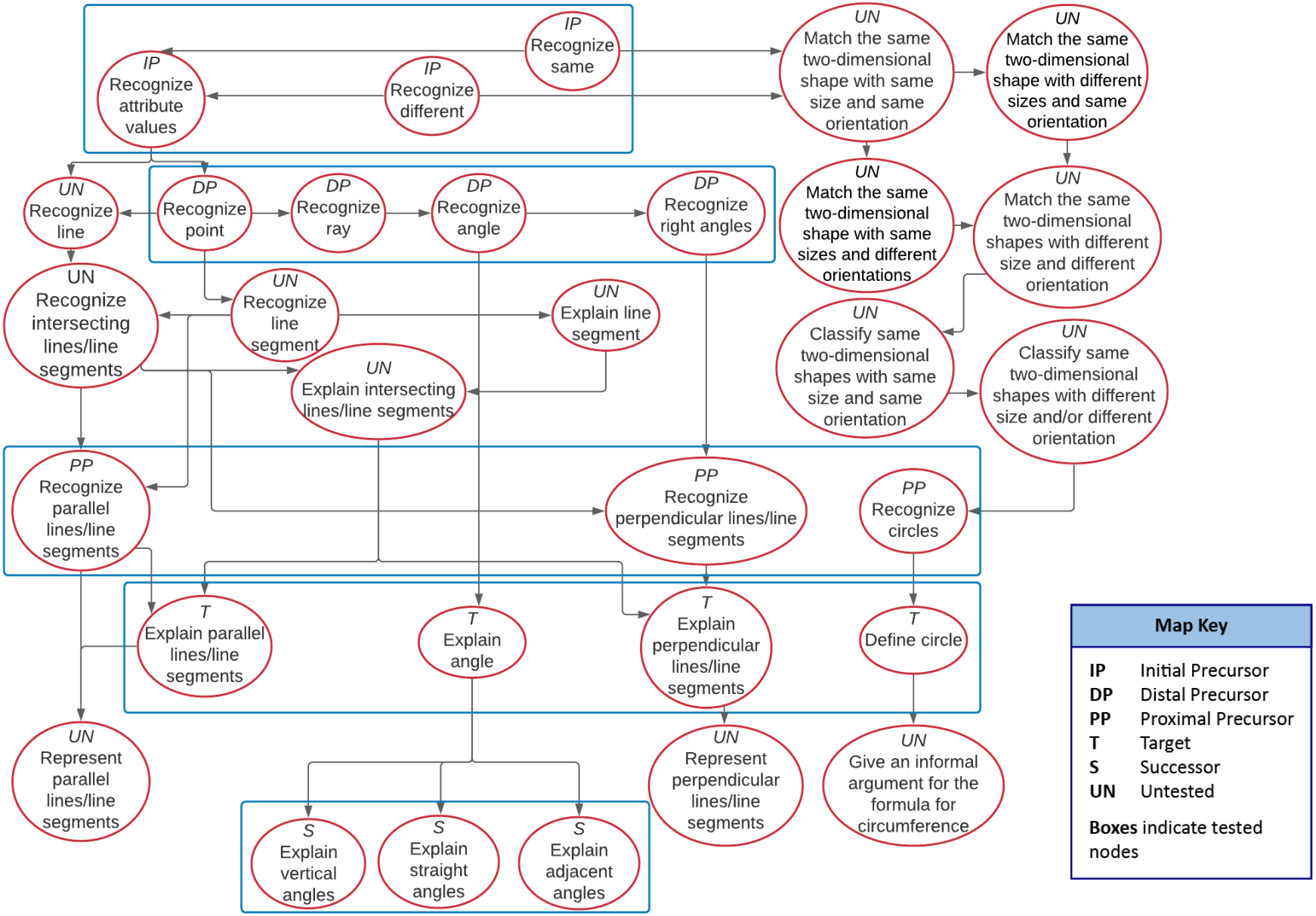
NOTE: Recognizing point should only be taught in the context of a lesson on lines, line segments, and angles.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.G.CO.1** Know the attributes of perpendicular lines, parallel lines, and line segments; angles; and circles.



# Mini-Map for M.EE.HS.G.CO.4-5

Subject: Mathematics
Geometry—Congruence (G.CO)
Grade: 10

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.G.CO.4-5** Given a geometric figure and a rotation, reflection, or translation of that figure, identify the components of the two figures that are congruent. | **M.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.<br>**M.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. |

## Linkage Level Descriptions

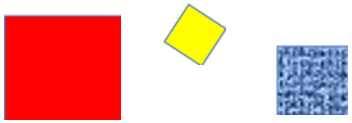
| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Recognize "same" as the object that shares all of the same attributes as other objects in a group. Recognize "different" as the object that shares some or none of the attributes as other objects in a group. | Match familiar shapes such as squares, rectangles, circles with shapes of the same size but with different orientations. Match familiar solids such as spheres, rectangular prisms, cubes, or pyramids with solids of the same size but with different orientations. | Identify the figure that is translated from the original view as a translation (slide), rotated from the original view as a rotation (turn), or reflected from the original view as a reflection (flip). Match a familiar shape, such as a square, circle, triangle, or rectangle, to a congruent figure with or without rotation or reflection. | Communicate understanding that two shapes are congruent if the second can be obtained from the first by a sequence of rotations, reflections, and translations. | Describe a sequence of transformations that would result in one figure being superimposed precisely over the other figure. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

Recognizing congruency requires a student to first recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., “this is a square” ). Then, provide multiple examples of the same shape so students can make comparisons, focusing student attention on the characteristics that make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as same or different.

NOTE: When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size).



***How is the Distal Precursor related to the Target?***

As students develop an understanding of same and different shapes, provide opportunities for students to match or group the same shapes based on the shape size (e.g., “this is a big square”, “this is a little square”). As students progress with identifying the size of shapes, the educator can begin to introduce different orientations of the shape.

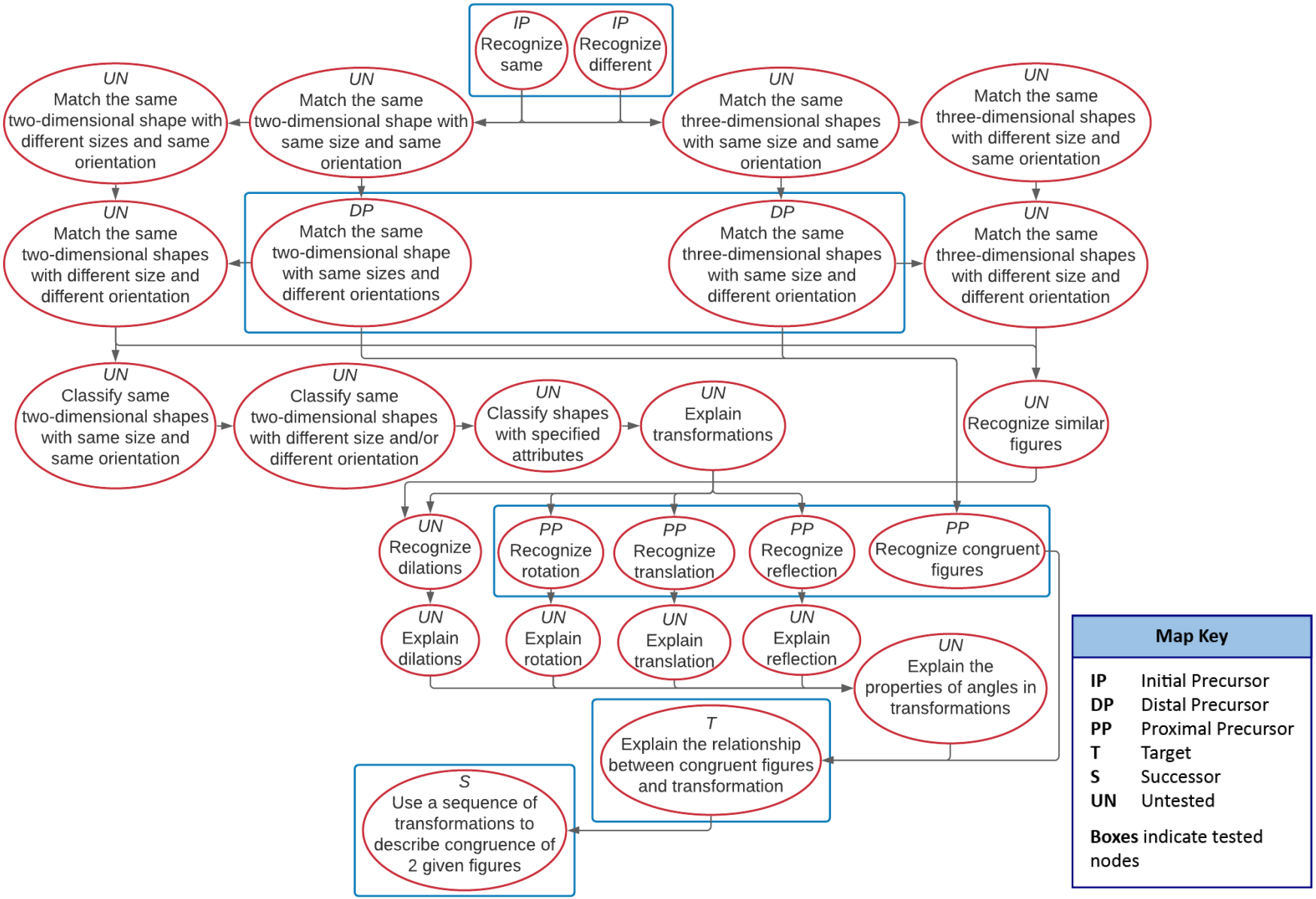
NOTE: As new attributes (e.g., size and orientation) are introduced, be sure to support the student in remembering that the attribute doesn't change the name of the shape.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.G.CO.4-5** Given a geometric figure and a rotation, reflection, or translation of that figure, identify the components of the two figures that are congruent.



# Mini-Map for M.EE.HS.G.CO.6-8

Subject: Mathematics
Geometry—Congruence (G.CO)
Grade: 11

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.G.CO.6-8** Identify corresponding congruent and similar parts of shapes. | **M.G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.<br>**M.G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Recognize "same" as the object that shares all of the same attributes as other objects in a group. Recognize "different" as the object that shares some or none of the attributes as other objects in a group. | Match two 3-dimensional shapes (e.g., spheres, rectangular prisms, cubes, pyramids) or 2-dimensional shapes (e.g., squares, rectangles, triangles) that are the same size and have either the same or different orientation. Match two 3-dimensional shapes (e.g., spheres, | Recognize two shapes that are congruent with or without rotation or reflection. Recognize two-dimensional and three-dimensional shapes that are similar. | Communicate understanding that congruent figures have the same shape and size and that similar figures have the same shape but different sizes. | Communicate understanding that two shapes are congruent if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Communicate understanding that two shapes are similar if the second can be obtained from the first by a sequence of dilations, |

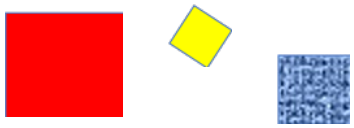
| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| | rectangular prisms, cubes, pyramids) or 2-dimensional shapes (e.g. squares, rectangles, triangles) that are different sizes and have either the same or different orientation. | | | rotations, reflections, or translations. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

Recognizing congruent and similar parts of a shape requires a student to first recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., "this is a square" ). Then, provide multiple examples of the same shape so students can make comparisons, focusing student attention on the characteristics that make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as same or different.

NOTE: When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size).



***How is the Distal Precursor related to the Target?***

As students develop an understanding of same and different shapes, provide opportunities for students to match or group the same shapes based on the shape size (e.g., "this is a big square", "this is a little square"). As students progress with identifying the size of shapes, the educator can begin to introduce different orientations of the shape as well as three-dimensional shapes.

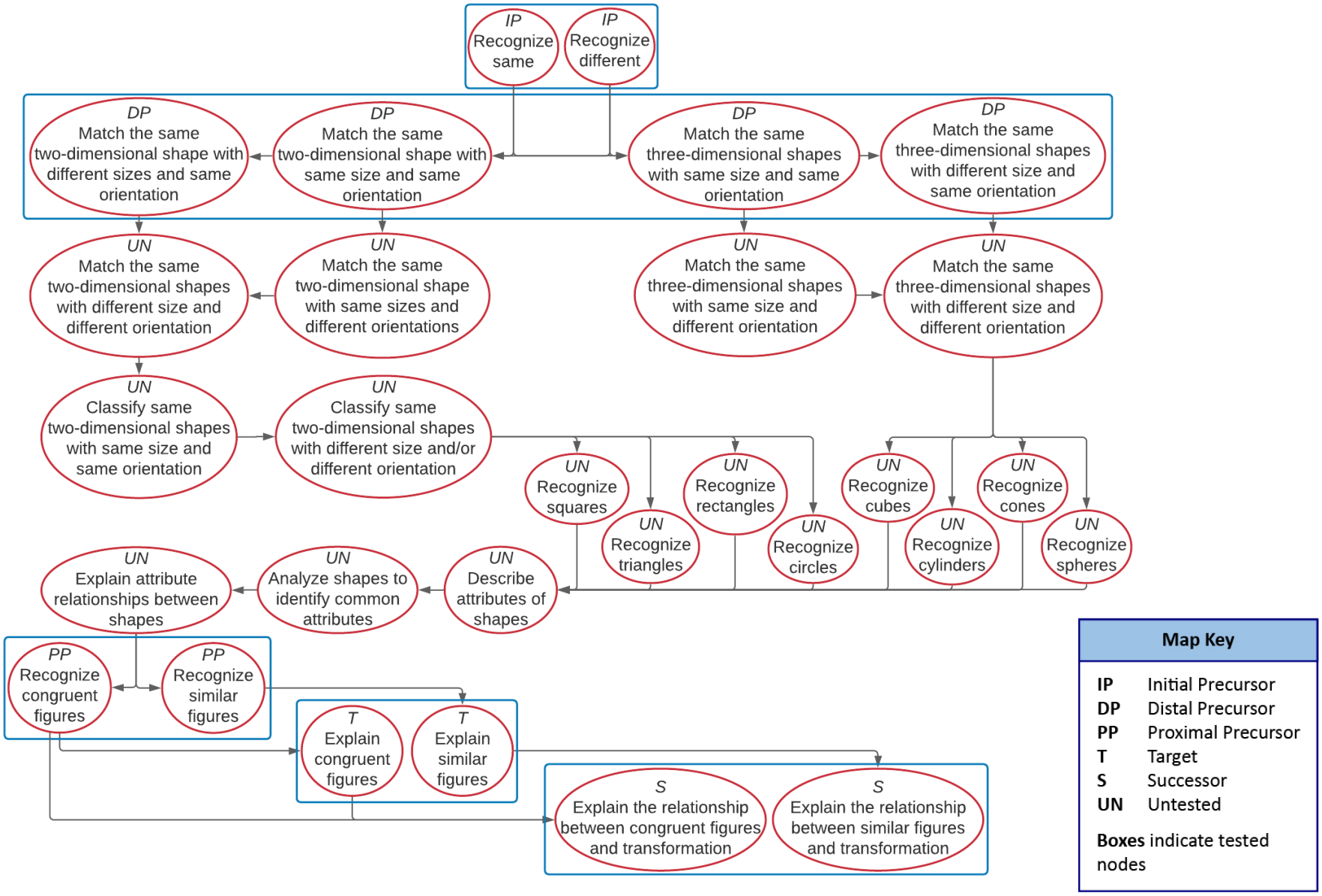
NOTE: As new attributes (e.g., size, orientation, three-dimensional) are introduced, be sure to support the student in remembering that the attribute doesn't change the name of the shape.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.G.CO.6-8** Identify corresponding congruent and similar parts of shapes.




**Map Key**

| | |
|---|---|
| **IP** | Initial Precursor |
| **DP** | Distal Precursor |
| **PP** | Proximal Precursor |
| **T** | Target |
| **S** | Successor |
| **UN** | Untested |

**Boxes** indicate tested nodes

# Mini-Map for M.EE.HS.G.MG.1-3

Subject: Mathematics
Geometry—Modeling with Geometry (G.MG)
Grade: 9

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.G.MG.1-3** Use properties of geometric shapes to describe real-life objects. | **M.G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).<br>**M.G.MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).<br>**M.G.MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). |

## Linkage Level Descriptions

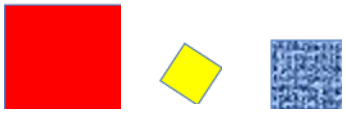
| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Recognize "same" as the object that shares all of the same attributes as other objects in a group. Recognize "different" as the object that shares some or none of the attributes as other objects in a group. | Match two 3-dimensional shapes (e.g., spheres, rectangular prisms, cubes, pyramids) or 2-dimensional shapes (e.g., squares, rectangles, triangles) that have the same orientation and either the same or different size. | Recognize a square, rectangle, circle, triangle, cube, cone, cylinder, and sphere. | Identify a real-world object using a geometrical shape (e.g., describing a roll of paper towels as a cylinder). | Create designs using paper clips, craft sticks, or straws to represent a given design problem (e.g., soccer field in the shape of a rectangle). |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to describe real-life objects, students must first recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., "this is a square" ). Then provide multiple examples of the same shape so students can make comparisons, focusing student attention on the characteristics that make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as same or different.

NOTE: When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size).



***How is the Distal Precursor related to the Target?***

As students develop an understanding of same and different shapes, provide opportunities for students to match or group the same shapes based on the shape size (e.g., "this is a big square", "this is a little square"). As students progress with identifying the size of shapes, the educator can begin to introduce different orientations of the shape as well as three-dimensional shapes.

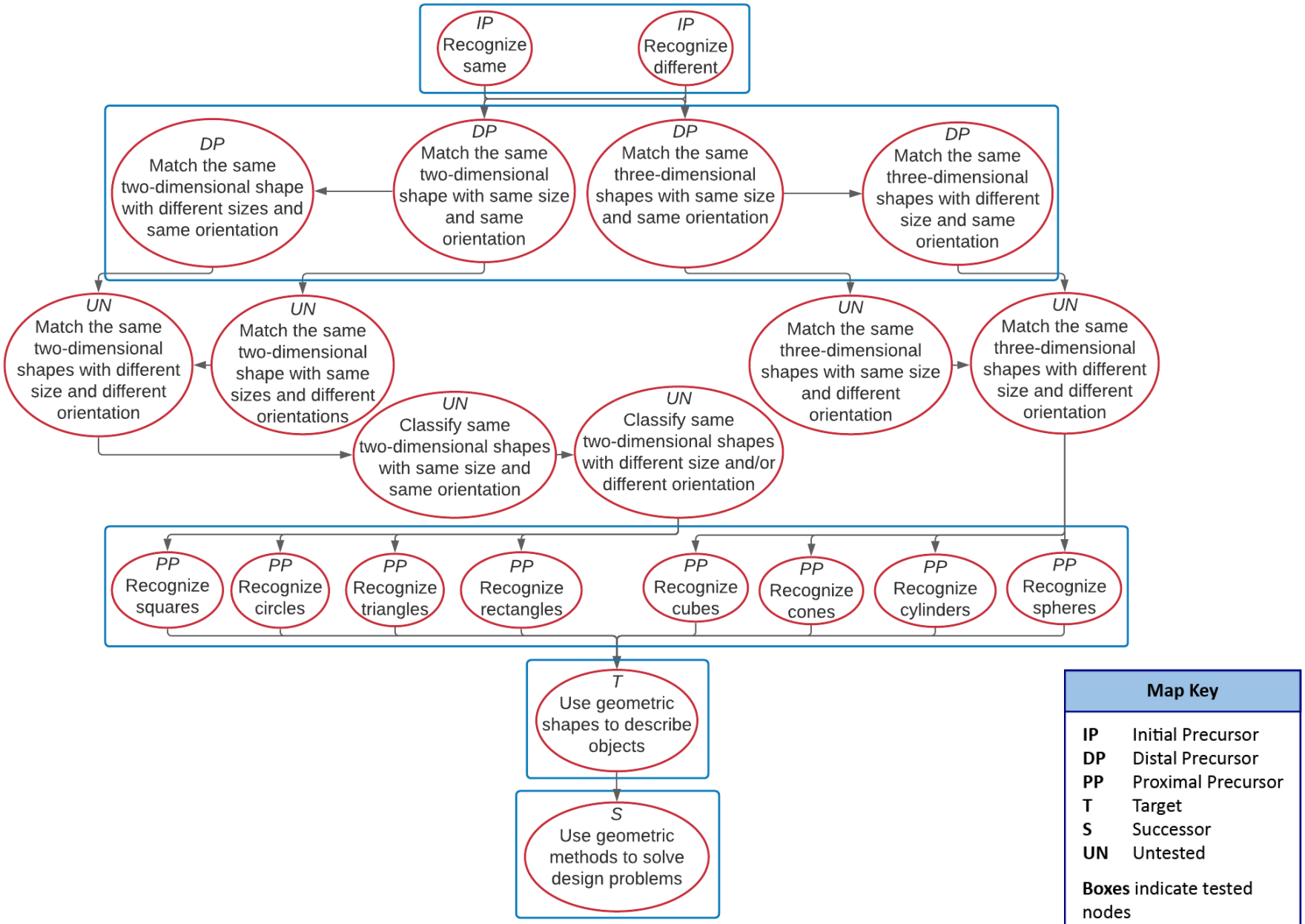
NOTE: As new attributes (e.g., size, orientation, three-dimensional) are introduced, be sure to support the student in remembering that the attribute doesn't change the name of the shape.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.G.MG.1-3** Use properties of geometric shapes to describe real-life objects.



# Mini-Map for M.EE.HS.G.GPE.7

Subject: Mathematics
Geometry—Expressing Geometric Properties with Equations (G.GPE)
Grade: 9

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.G.GPE.7** Find perimeters and areas of squares and rectangles to solve real-world problems. | **M.G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula). |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Recognize attributes or characteristics of an object, such as color, orientation, length, width, and weight. | Recognize measurable attributes (e.g., height, depth, diameter, weight) and differentiate them from non-measurable attributes (e.g., color or orientation). | Calculate the perimeter of a polygon by adding up all the side lengths. Calculate the area of a square or rectangle by counting the number of square units drawn to cover the area. | Solve real-world problems by determining the area of a square or a rectangle. Solve real-world problems by calculating the perimeter of polygons. | Represent a real-life situation involving the perimeter of a polygon or the area of a polygon using expressions, equations, diagrams, or graphs. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to find the perimeter and area of a shape, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “this is a circle, so it does not have sides”, “this is a rectangle, so it has four sides”), and the student observes, feels, or otherwise interacts with the shapes.

***How is the Distal Precursor related to the Target?***

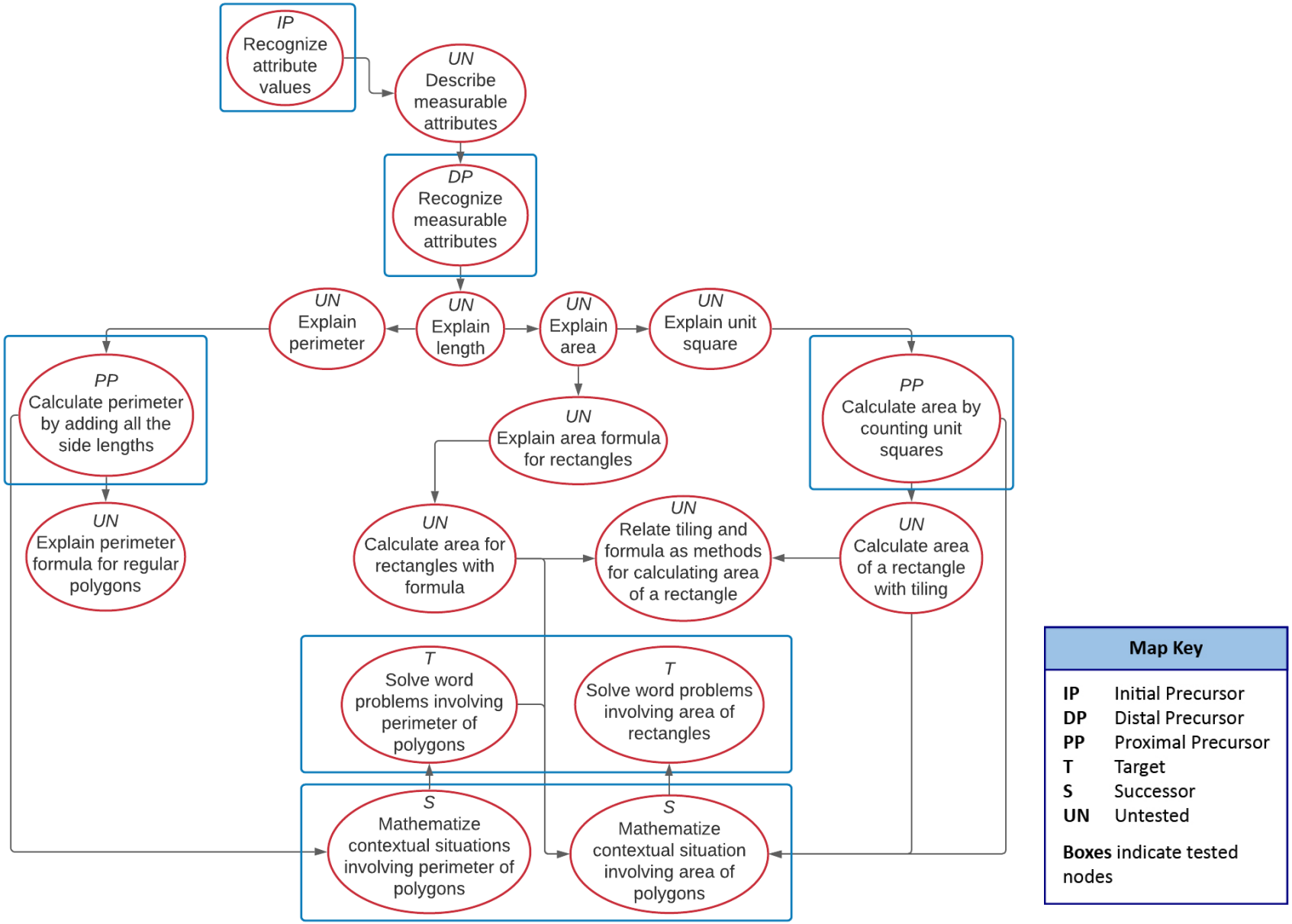
As students develop their attention to objects and notice the difference between objects, they will begin working on recognizing and describing measurable attributes. Students need lots of experience making direct comparisons between objects. Educators should use and demonstrate the meaning of comparison words (e.g., big/small, tall/short, longer/shorter). While students do not need to say them, they do need to learn their meaning.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.G.GPE.7** Find perimeters and areas of squares and rectangles to solve real-world problems.



# Mini-Map for M.EE.HS.N.Q.1-3

Subject: Mathematics
Number and Quantity—Quantities (N.Q)
Grade: 10

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.N.Q.1-3** Express quantities to the appropriate precision of measurement. | **M.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.<br>**M.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.<br>**M.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Without counting each object, identify the number of objects in a set (up to four). | Round decimals to any place by using standard rounding off rules (e.g., round up when the digit in the tenths place is 5 or greater, and round down when the digit in the tenths place is less than 5). For example, round 8.5 to 9.0. | Solve word problems involving addition, subtraction, and multiplication of rational numbers. (Limit decimal answers to hundredths.) | Report answers to numerical problems involving decimals with a degree of precision appropriate to the problem context (e.g., report the area of a rectangle with sides 6.5 cm and 4.32 cm as 15.1 $cm^2$ rather than 15.12 $cm^2$). | Solve multi-step, real-world problems involving rational numbers, limiting all the numbers in the problem to whole numbers and decimals to the hundredths (e.g., Miguel earns $8.75 each day for 5 days. He spends $18.80 on a game. How much money does Miguel have left?). |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

To express quantities with precision, students first need to know number names, the count sequence, one-to-one correspondence, and have cardinality. These procedures and concepts develop through many experiences in early counting. Perceptual subitizing happens when the student is able to name the amount (1-3 items) without actually counting them. For example when an educator asks the student to get their shoes and asks, "How many shoes do you have?" The student would reply, "two," without using the count sequence of one, two. This only happens when students have been given many experiences counting small numbers with many different contexts and materials.

NOTE: Students who are blind will learn to use tactile enumeration for 1-3 items.

***How is the Distal Precursor related to the Target?***

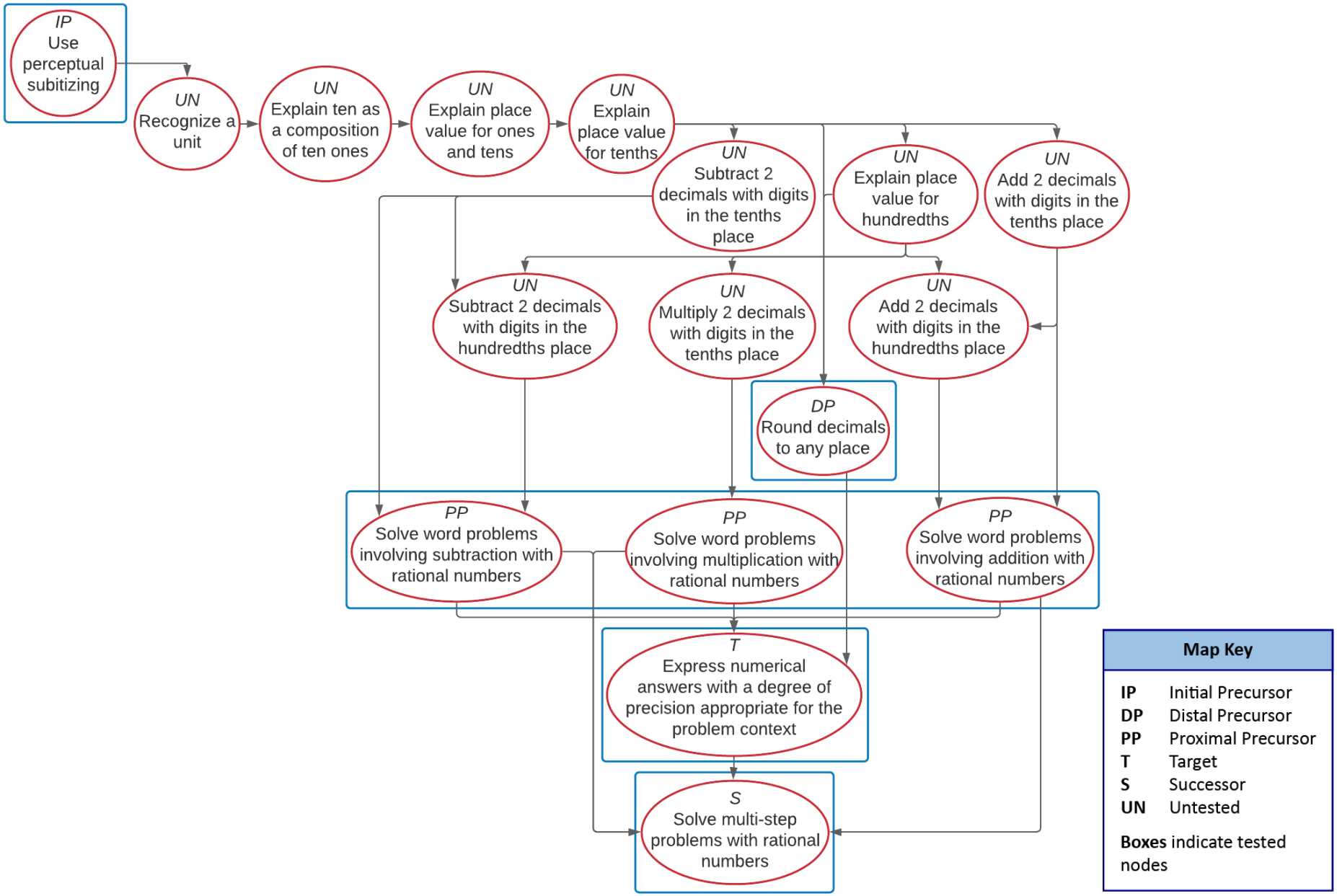
As students continue to gain experience in counting, educators will introduce the concept that 10 can be grouped into one unit. Educators will use models that help students perceive a group of 10 and some more (e.g., bundles, ten-frames, number line, arrays, etc.). Teen numbers are an important part of understanding this concept. Additionally, educators provide students experience working with money values (e.g., $2.42, $0.67, $5.94) and learning how to round up to the nearest dollar (e.g., $2.42 rounds to $3.00) or tenths place (e.g., $0.67 rounds to $0.70) or ones place (e.g., $5.94 rounds to $5.95). Students should also have experience with rounding down, but not in the context of money (e.g., 0.73 rounds to 70).

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.N.Q.1-3** Express quantities to the appropriate precision of measurement.



# Mini-Map for M.EE.HS.S.ID.1-2

Subject: Mathematics
Statistics and Probability—Interpreting Categorical and Quantitative Data (S.ID)
Grade: 10

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.S.ID.1-2** Given data, construct a simple graph (line, pie, bar, or picture) or table, and interpret the data. | **M.S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).<br>**M.S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Arrange objects in a specific order (e.g., smallest to largest). Group objects by some attribute value (e.g., shape, size, texture, numerical pattern). | Recognize the structure of bar, picture, line graphs, and pie charts, such as the title and labels for the *x*- and *y*-axes. Understand that bars are used to display data on bar graphs. Understand that pictures, symbols, or geometric figures are used to display data on picture graphs. Understand that points joined by a line are used to represent data on | Answer questions by lifting information from a bar graph, picture graph, or line plot and understand the information represented on the graph (e.g., in the graph representing students' favorite ice cream, how many students like strawberry ice cream? How many students like chocolate ice cream?). | Represent data on bar graphs, picture graphs, line graphs, and pie charts. Use bar graphs, picture graphs, line graphs, and pie charts to answer questions (e.g., how many, most, least) that require interpretation and integration of information presented in the graph. | Draw inferences or make predictions by interpreting information presented on a bar graph, picture graph, line graph, or pie chart (e.g., on the bar graph representing the number of pizzas required for a certain number of people, predict the number of pizzas needed for 20 people). |

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| | line graphs, and sectors are used to represent data on pie charts. | | | |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to represent and use data, students begin by learning to recognize what is the same and different between familiar items such as color, shape, quantity, size, texture, and pattern. Educators should take care to use words that describe (e.g., more, less, red circle, same, different) while defining and demonstrating their meaning. While students do not need to say these words, they do need to learn the meanings. Students will also begin to group two or more items in the same set based on an attribute (e.g., two CDs, bumpy balls and bumpy gravel, red rectangles). As the students group two or more items, the educator will demonstrate the representation in graphs and charts and encourage students to actively participate in their creation.

***How is the Distal Precursor related to the Target?***

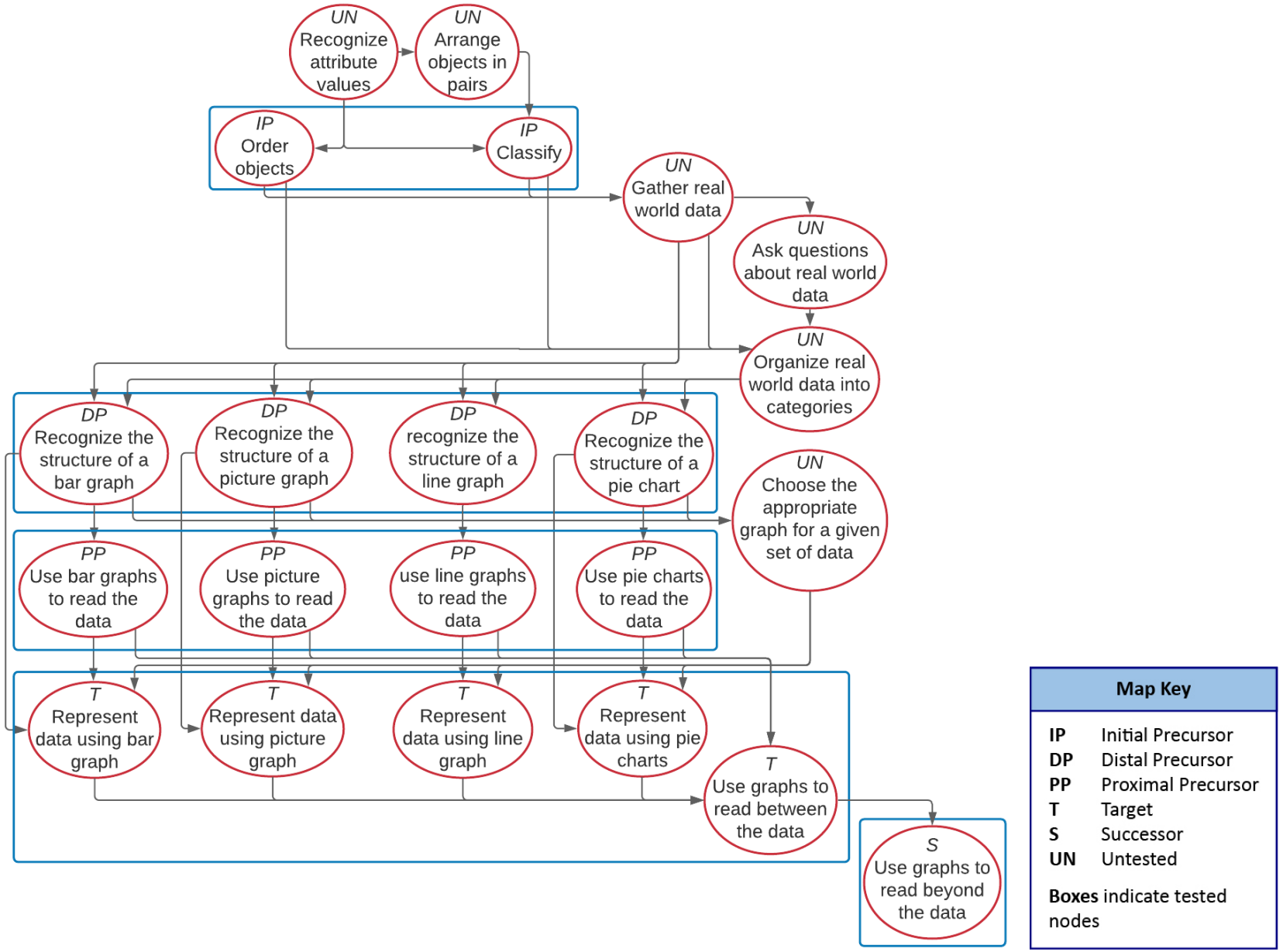
Students actively participate in the creation of bar graphs, picture graphs, line graphs, and pie charts by placing representations for each response to the research question.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.S.ID.1-2** Given data, construct a simple graph (line, pie, bar, or picture) or table, and interpret the data.



# Mini-Map for M.EE.HS.S.ID.3

Subject: Mathematics
Statistics and Probability—Interpreting Categorical and Quantitative Data (S.ID)
Grade: 11

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.S.ID.3** Interpret general trends on a graph or chart. | **M.S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Arrange objects in a specific order (e.g., smallest to largest). Group objects by some attribute value (e.g., shape, size, texture, numerical pattern). | Recognize the structure of bar graphs, picture graphs, line plots, and pie charts, such as the title and labels for the *x*- and *y*-axes. Understand that bars are used to display data on bar graphs. Understand that pictures, symbols, or geometric figures are used to display data on picture graphs. Understand that on a line plot, "x" is used to represent the data values, and sectors are | Recognize symmetric distribution, outliers, and peaks in a data distribution shown graphically. Recognize data values substantially larger or smaller than the other values as outliers. Recognize peaks as data values that most frequently occur. Recognize symmetric distribution as distributions where the left- and right-hand sides of the distributions are roughly equal. | Analyze the overall shape of the data distribution and communicate whether the distribution is symmetric, has outlier(s), or peaks. Draw inferences by interpreting general trends on a graph or chart. | Draw inferences by comparing the shape and spread of two data sets (e.g., the student compares the peaks of two sets of data, height of soccer players and height of basketball players, to communicate that basketball players are, in general, taller than soccer players). |

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| | used to represent data on pie charts. | Recognize whether a set of scores is spread-out or grouped together (variability). | | |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to construct a graph, students begin by learning to recognize what is the same and different between familiar items, such as color, shape, quantity, size, texture, and pattern. Educators should take care to use words that describe (e.g., more, less, red circle, same, different) while defining and demonstrating their meaning. While students do not need to say these words, they do need to learn the meanings. Students will also begin to group two or more items in the same set based on an attribute (e.g., two CDs, bumpy balls and bumpy gravel, red rectangles). As the students group two or more items, the educator will demonstrate the representation in graphs and charts and encourage students to actively participate in their creation.

***How is the Distal Precursor related to the Target?***

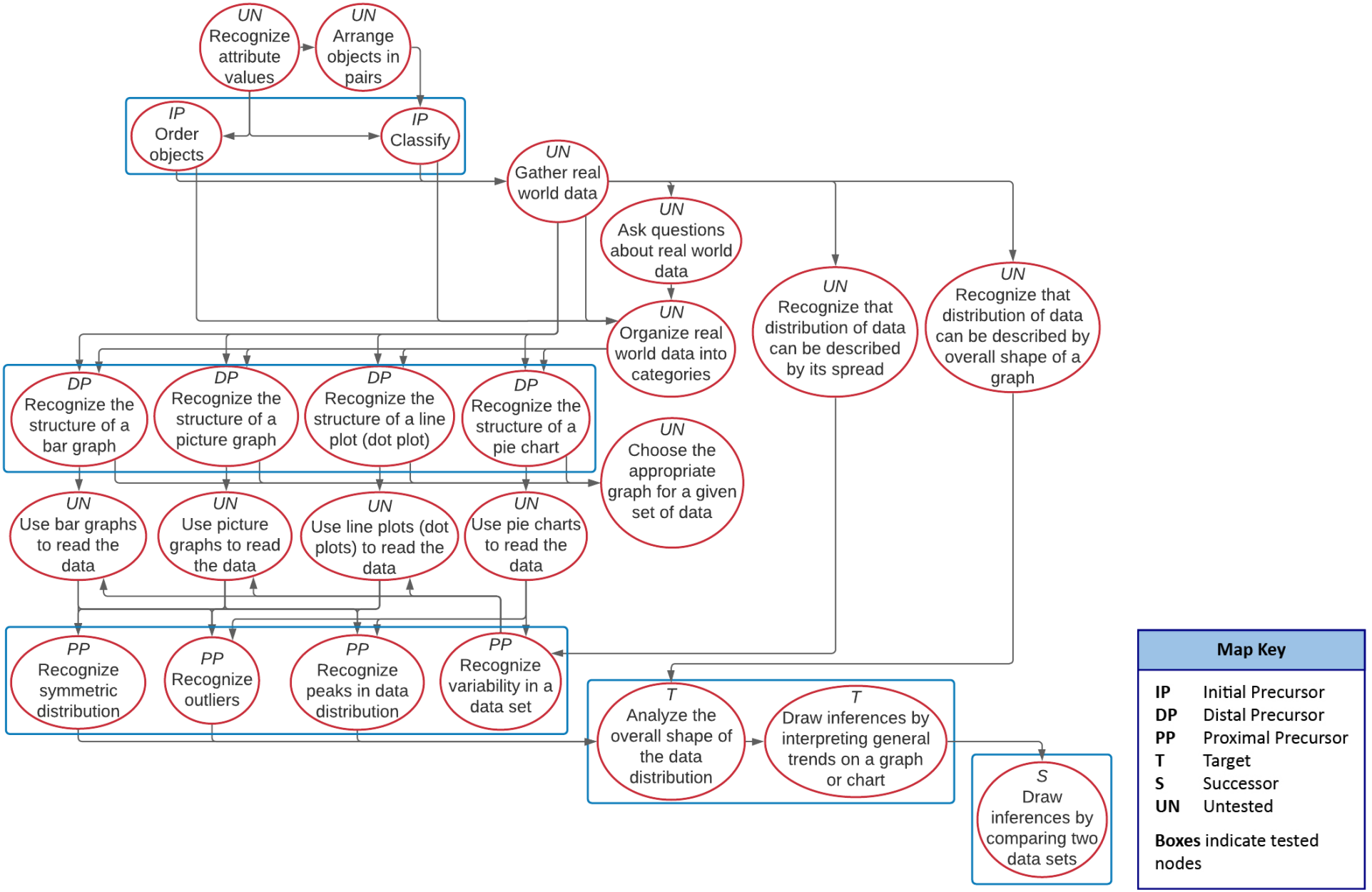
Students actively participate in the creation of bar graphs, picture graphs, line graphs, and pie charts by placing representations for each response to the research question.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.S.ID.3** Interpret general trends on a graph or chart.



# Mini-Map for M.EE.HS.S.ID.4

Subject: Mathematics
Statistics and Probability—Interpreting Categorical and Quantitative Data (S.ID)
Grade: 10

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.S.ID.4** Calculate the mean of a given data set (limit the number of data points to fewer than five). | **M.S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Recognize attributes or characteristics of an object, such as color, orientation, length, width, and weight. | Group objects by some attribute value (e.g., shape, size, texture, numerical pattern). | Communicate the number of observations for a given set of data [e.g., the number of observations in a data set {2, 5, 8, 10, 15, 4, 8} is 7]. | Calculate mean by dividing the sum of all data by the number of observations. | Summarize data by calculating the mode or median [e.g., the mode for the data set {1, 2, 5, 6, 2, 3, 4, 2, 2} is 2]. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to calculate the mean of a data set, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “this is a circle since it does not have any sides”, “two fidgets are big and two fidgets are small”), and the student observes, feels, or otherwise interacts with the shapes. Students also work on counting small units, recognizing that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. As educators present a set, label it (e.g., two balls, one bear, three blocks), count the items, label it again, and encourage students to use numbers to label and count the separate sets.

***How is the Distal Precursor related to the Target?***

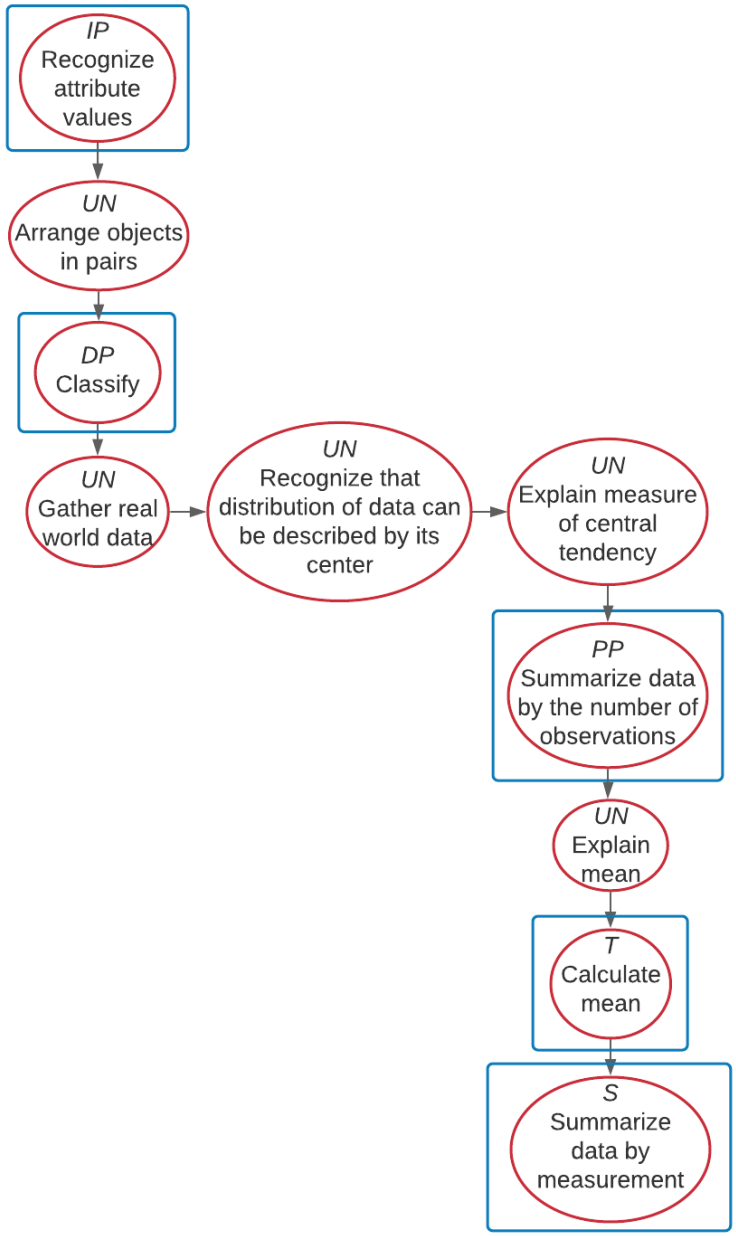
As students develop their ability to attend to the details of an object and to count objects, educators provide many opportunities for students to classify (group) items based on their size (e.g., compare two or more items and determine which is larger or smaller), amount (e.g., numbers larger or smaller than a given number), and distance between numbers (e.g., skip counting by 2, 5, or 10). Educators should also take care to use attribute words when defining and demonstrating grouping items. While students do not need to say these words, they do need to learn the meanings.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.S.ID.4** Calculate the mean of a given data set (limit the number of data points to fewer than five).




| Map Key | |
|---|---|
| **IP** | Initial Precursor |
| **DP** | Distal Precursor |
| **PP** | Proximal Precursor |
| **T** | Target |
| **S** | Successor |
| **UN** | Untested |

**Boxes** indicate tested nodes

# Mini-Map for M.EE.HS.A.CED.1

Subject: Mathematics
Algebra—Creating Equations (A.CED)
Grade: 10

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.A.CED.1** Create an equation involving one operation with one variable, and use it to solve a real-world problem. | **M.A.CED.1** Create equations and inequalities in one variable, and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. |

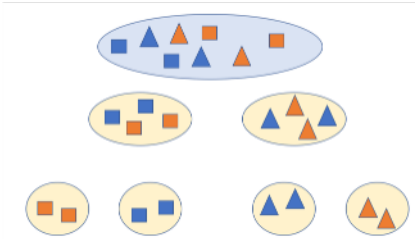
## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Combine two or more sets of objects or numbers to form a new set. Divide a set of 10 or fewer objects into two or more distinct subsets (e.g., dividing a set containing 10 objects into two subsets containing 4 and 6 objects). | Represent addition, subtraction, multiplication, or division word problems or models with equations (e.g., representing 6 marbles plus 2 marbles equal 8 marbles as 6 + 2 = 8 marbles). | Represent expressions using variables and numbers (e.g., express subtract *k* from 12 as 12 - *k*). Recognize that the unknown quantity in an equation is represented using a symbol or letter (e.g., 5 + *b* = 8). | Solve real-world problems with non-negative rational numbers by representing the situation with a mathematical equation (e.g., Mark has 3.5 inches of string. Mark gets 1 more inch of string. Which equation shows how much string Mark has all together? 3.5 + 1 = *x*). | Solve equations with non-negative rational numbers involving addition, subtraction, multiplication, or division operations in one variable (e.g., 8.4 + *x* = 17.56). |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

Representing and solving equations requires a student to count small units, recognizing that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numbers to label and count the separate sets. The general goal is to explore how the set changes when items are separated out (partitioned) or combined.



***How is the Distal Precursor related to the Target?***

As students begin to understand labeling and counting small sets, they begin to use the number sequence and become more adept at tracking individual objects. Work on this skill using a variety of sets, labeling and counting the sets, and moving items in and out of the sets, labeling and counting the set again. Additionally, the educators will pair those sets with the symbolic representations for addition, subtraction, multiplication, and division (e.g., $3 \times 2 = ?$, $3 - 2 = ?$).

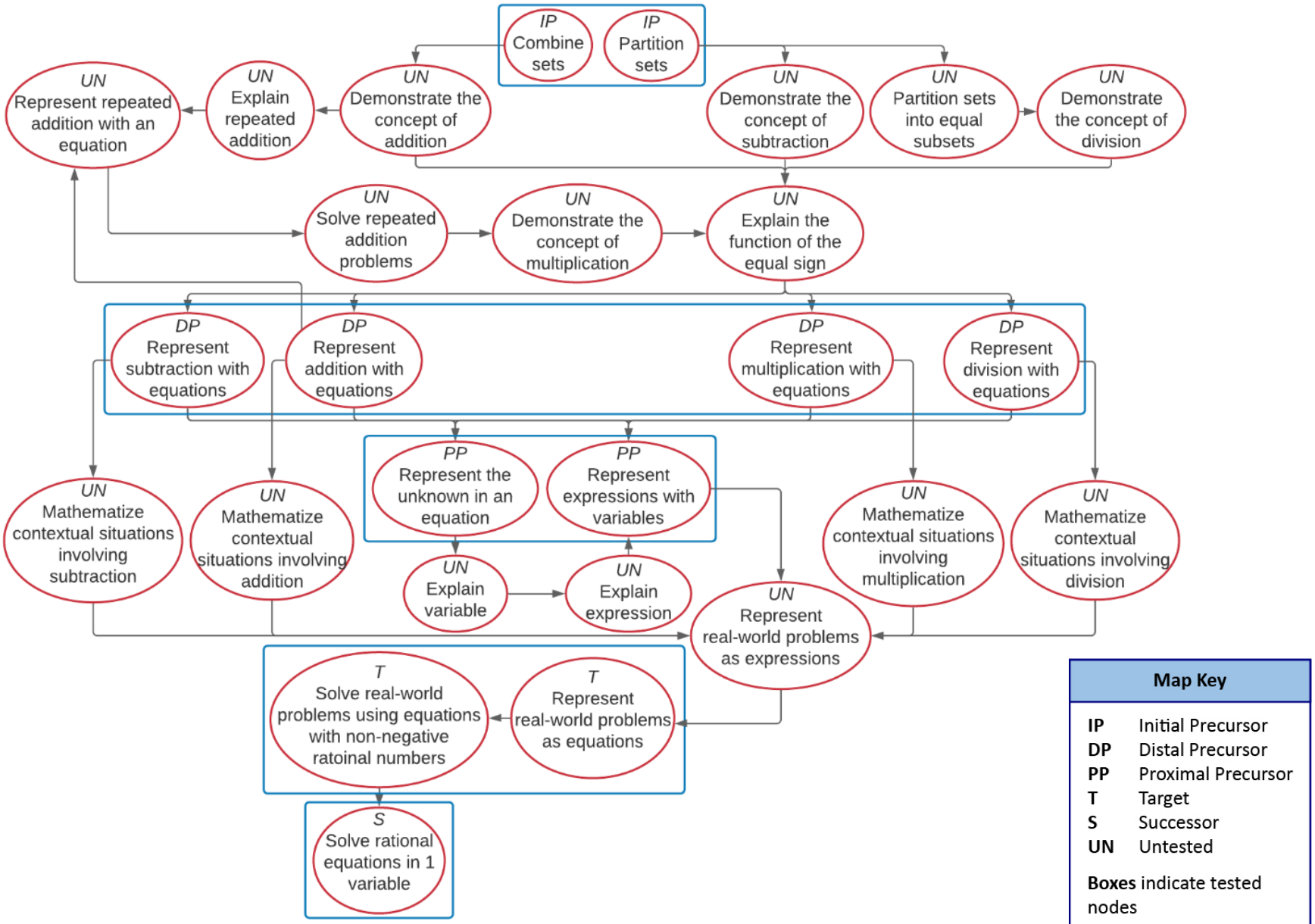
NOTE: Educators can work on the Distal Precursor level using the sets of numbers that students working at the Target level are working with.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.A.CED.1** Create an equation involving one operation with one variable, and use it to solve a real-world problem.



# Mini-Map for M.EE.HS.A.CED.2-4

Subject: Mathematics
Algebra—Creating Equations (A.CED)
Grade: 10

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.A.CED.2-4** Solve one-step inequalities. | **M.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.<br>**M.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.<br>**M.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. |

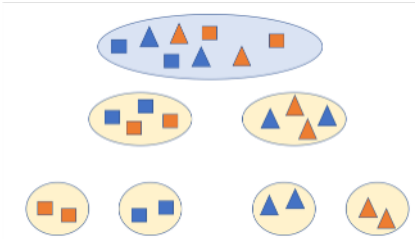
## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Combine two or more sets of objects or numbers to form a new set. Divide a set of 10 or fewer objects into two or more distinct subsets (e.g., dividing a set containing 10 objects into two subsets containing 4 and 6 objects). | Represent addition, subtraction, multiplication, or division word problems or models with equations (e.g., representing 6 marbles plus 2 marbles equal 8 marbles as 6 + 2 = 8 marbles). | Solve linear equations involving addition, subtraction, multiplication, or division operations in one variable (e.g., 8.4 + $x$ = 17.56). | Solve linear inequalities in one variable (e.g., $x$ + 7 < 14), and represent solutions to inequalities on a number line. | Explain a solution to a linear inequality in one variable (e.g., x < 8 means that x takes all the values less than 8; i.e., 7, 6, 5...). |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

The knowledge needed to solve one-step inequalities requires students to manipulate sets (i.e., combining and separating or partitioning). Provide students many opportunities to take a set of objects (e.g., tiles, linking cubes, buttons) and separate them based on a given characteristic (e.g., shape, color, size) into two distinct sets, then separate them again based on another characteristic. Guide students to notice how the set size changes each time you combine or partition the sets.



***How is the Distal Precursor related to the Target?***

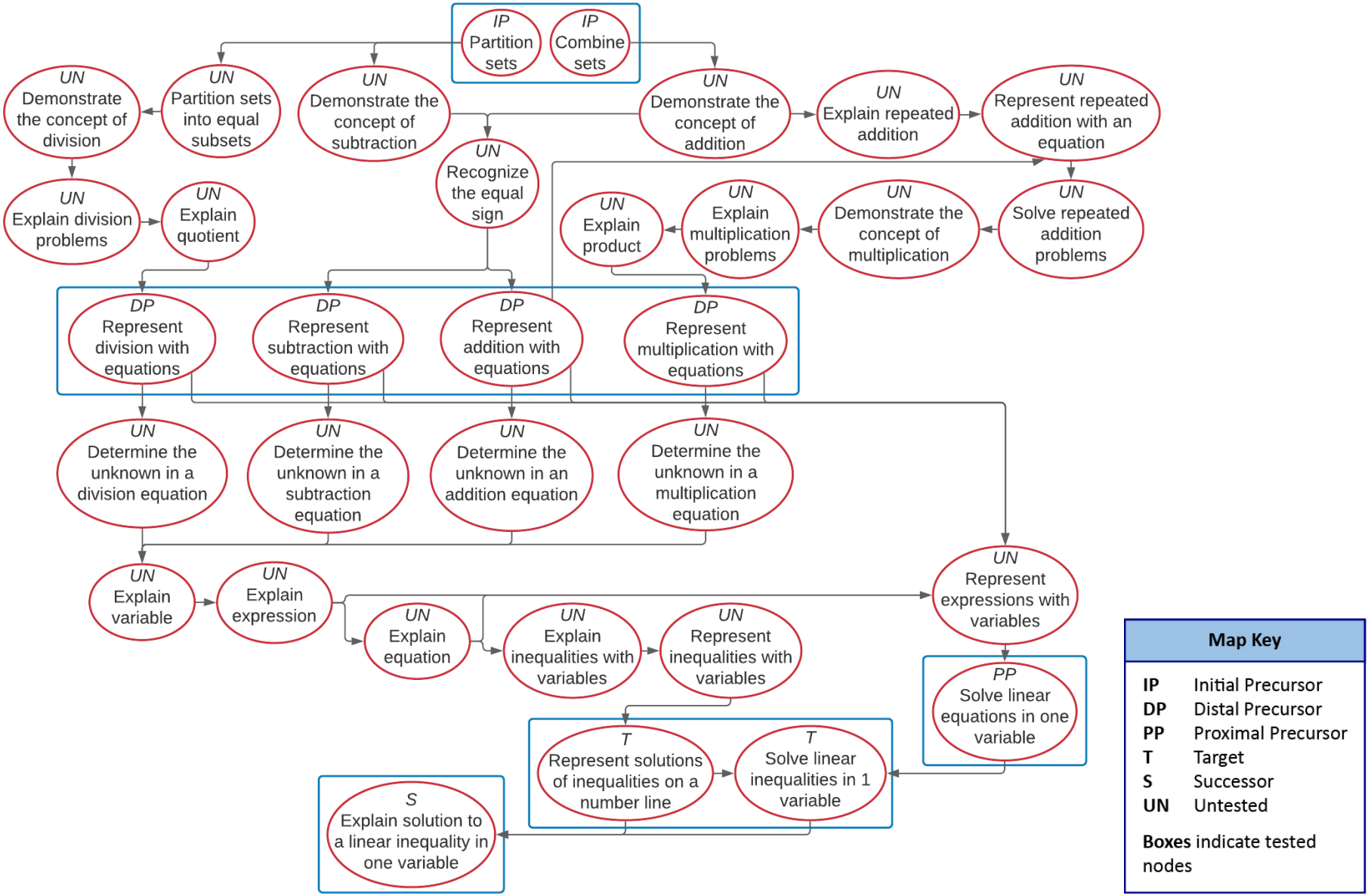
As students begin to understand labeling and counting sets, they begin to use the number sequence and become more adept at tracking individual objects. Work on this skill using a variety of sets, labeling and counting the sets, and moving items in and out of the sets, labeling and counting the set again. Additionally, the educators will pair those sets with the symbolic representations for addition, subtraction, multiplication, and division (e.g., 3 + 2 = ?, 3 × 2 =?).

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

**M.EE.HS.A.CED.2-4** Solve one-step inequalities.



# Mini-Map for M.EE.HS.A.SSE.1

Subject: Mathematics
Algebra—Seeing Structure in Expressions (A.SSE)
Grade: 9

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.A.SSE.1** Identify an algebraic expression involving one arithmetic operation to represent a real-world problem. | **M.A.SSE.1** Interpret expressions that represent a quantity in terms of its context. |

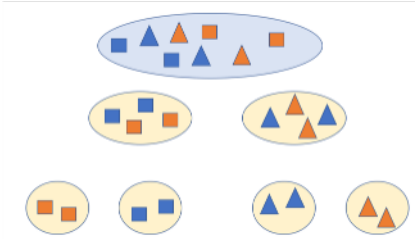
## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Combine two or more sets of objects or numbers to form a new set. Divide a set of 10 or fewer objects into two or more distinct subsets (e.g., dividing a set containing 10 objects into two subsets containing 4 and 6 objects). | Represent addition, subtraction, multiplication, or division word problems or models with equations (e.g., representing 6 marbles plus 2 marbles equal 8 marbles as 6 + 2 = 8 marbles). | Represent expressions using variables and numbers (e.g., express subtract *k* from 12 as 12 - *k*). Recognize that the unknown quantity in an equation is represented using a symbol or letter (e.g., 5 + *b* = 8). | Represent real-world problems using expressions and equations. | Solve real-world problems by representing problems using equations with non-negative rational numbers. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

The knowledge needed to represent equations requires students to manipulate sets (i.e., combining and separating or partitioning). Provide students many opportunities to take a set of objects (e.g., tiles, linking cubes, buttons) and separate them based on a given characteristic (e.g., shape, color, size) into two distinct sets, then separate them again based on another characteristic. Guide students to notice how the set size changes each time you combine or partition the sets.



***How is the Distal Precursor related to the Target?***

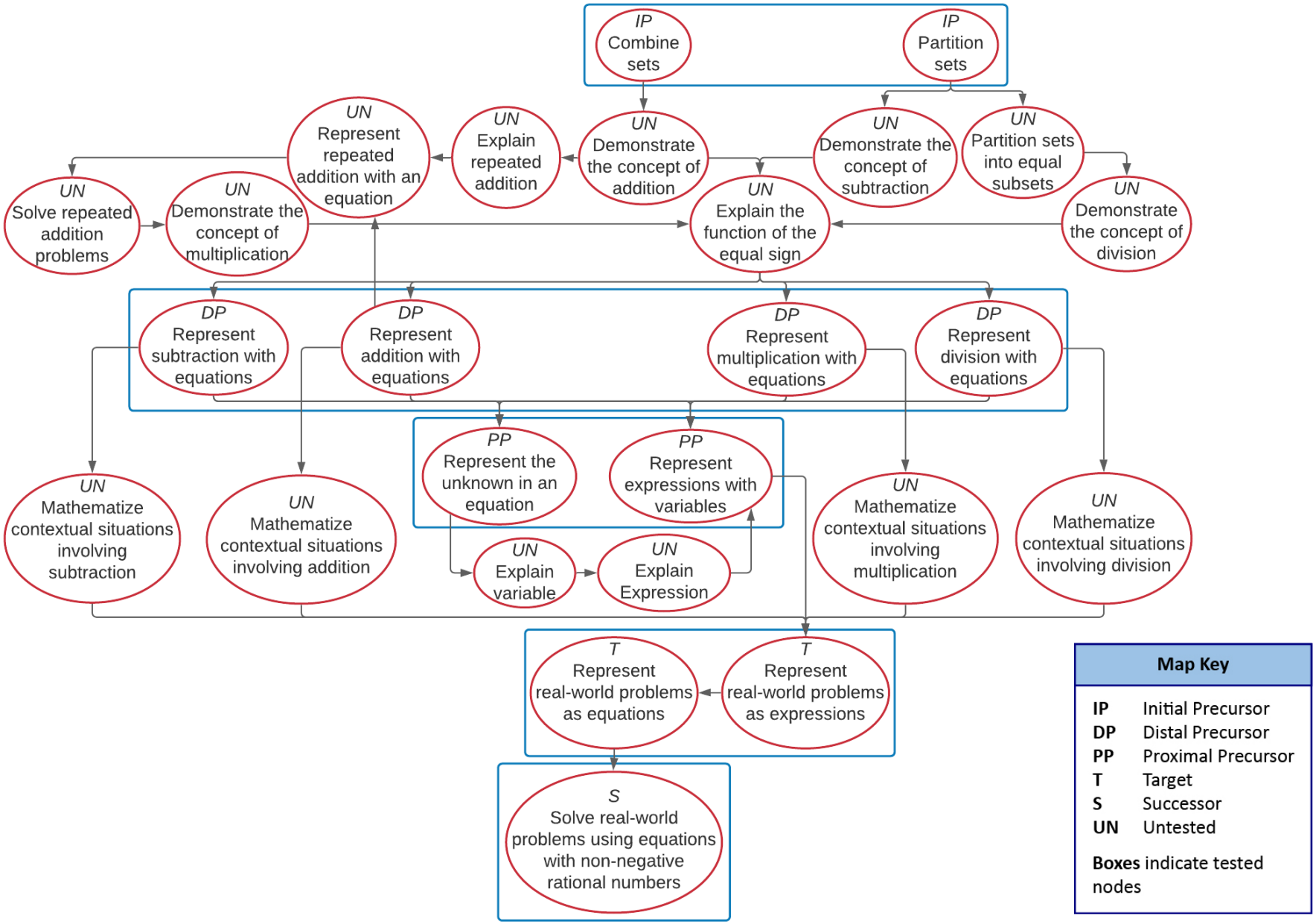
As students begin to understand labeling and counting sets, they begin to use the number sequence and become more adept at tracking individual objects. Work on this skill using a variety of sets, labeling and counting the sets, and moving items in and out of the sets, labeling and counting the set again. Additionally, the educators will pair those sets with the symbolic representations for addition, subtraction, multiplication, and division (e.g., 3 + 2 = ?, 3 × 2 =?).

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

**M.EE.HS.A.SSE.1** Identify an algebraic expression involving one arithmetic operation to represent a real-world problem.



# Mini-Map for M.EE.HS.A.SSE.3

Subject: Mathematics
Algebra—Seeing Structure in Expressions (A.SSE)
Grade: 9

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.A.SSE.3** Solve simple algebraic equations with one variable using multiplication and division. | **M.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. |

## Linkage Level Descriptions

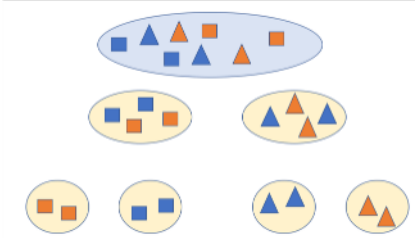
| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Combine two or more sets of objects or numbers to form a new set. Divide a set of 10 or fewer objects into two or more distinct subsets (e.g., dividing a set containing 10 objects into two subsets containing 4 and 6 objects). | Demonstrate multiplication by combining multiple sets containing the same number of objects. Communicate understanding that the number of sets times the number of objects in each set equals the total number of objects. Demonstrate understanding of division by splitting a set into an equal number of subsets and communicating the quotient as the number of equal subsets (e.g., a set consisting of 15 | Determine the unknown factor or product in an equation involving multiplication (e.g., 6 x 7 = ?) . Determine the unknown divisor, dividend, or quotient in an equation involving division (e.g., 24 ÷ 4 = ?). | Solve linear equations with non-negative rational numbers involving addition or subtraction operations in one variable (e.g., 3.3 + $x$ = 8.9). | Solve linear inequalities in one variable (e.g., 2.5$x$ > 100.25). |

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| | objects has three subsets, each containing 5 objects). | | | |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

The knowledge needed to represent equations requires students to manipulate sets (i.e., combining and separating or partitioning). Provide students many opportunities to take a set of objects (e.g., tiles, linking cubes, buttons) and separate them based on a given characteristic (e.g., shape, color, size) into two distinct sets, then separate them again based on another characteristic. Guide students to notice how the set size changes each time you combine or partition the sets.



***How is the Distal Precursor related to the Target?***

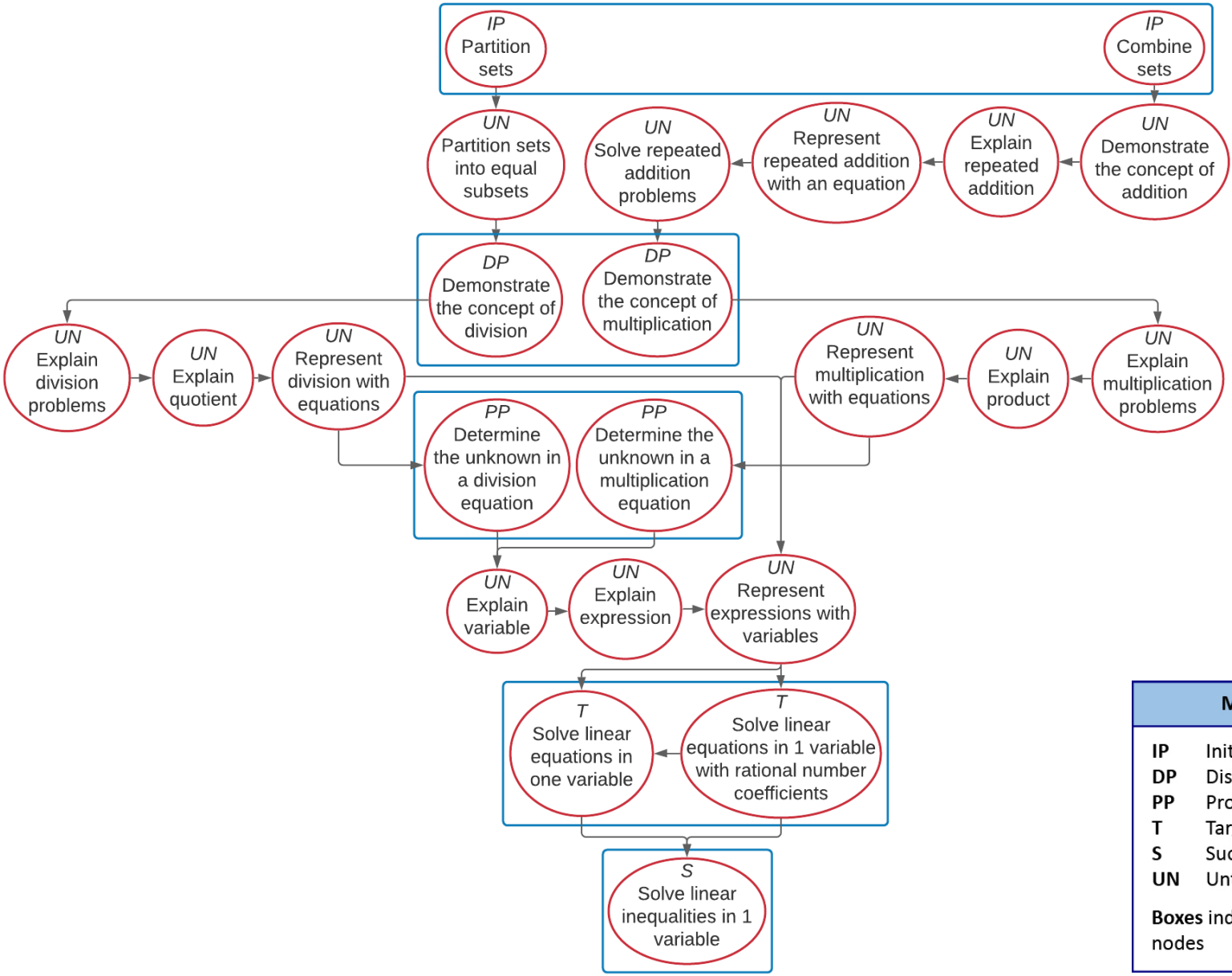
As students' understanding of labeling and counting sets develops, they will begin working on adding items to a set and combining sets to create a new set. Additionally, students will work on developing an understanding of equal shares by actively participating in one-to-one distribution of objects to person (e.g., giving each person in the group two pencils), objects to objects (e.g., given four counters, they would line up four more counters in front of or on top of the first set), and objects to available space (e.g., given three chairs at a table, the student places a cup on the table for each available chair). Students should also experience dividing a whole into equal shares (e.g., having 15 counters and 3 people in the group, give one to each person until there are no more, then count how many each person received).

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

[Link to Text-Only Map]

**M.EE.HS.A.SSE.3** Solve simple algebraic equations with one variable using multiplication and division.



# Mini-Map for M.EE.HS.A.REI.10-12

Subject: Mathematics
Algebra—Reasoning with Equations and Inequalities (A.REI)
Grade: 10

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.A.REI.10-12** Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas. | **M.A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).<br>**M.A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.<br>**M.A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Form a pair by arranging objects in specific order (e.g., putting together a pencil and a ruler, where the pencil is placed first and the | Communicate understanding that a coordinate pair (ordered pair) is a set of numbers used to show a position on a graph. The first number, "*x*," or the *x*-coordinate in the | Recognize covariation as the pattern in which two variables or quantities change together. Recognize the direction in which two variables change together (e.g., as *x* | Examine the shape of a linear function graph to recognize the relationship between two varying quantities (e.g., increasing, decreasing, constant). Use the understanding | Find solutions to real-world problems by interpreting linear function graphs. |

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| ruler is placed next to it). | coordinate pair (*x*, *y*), represents *x* units left or right on the *x*-axis. The second number, "*y*," or the *y*-coordinate, represents *y* units up or down on the *y*-axis [e.g., (4, 8) represents 4 units right on the *x*-axis and 8 units up on the *y*-axis]. | increases, *y* decreases). Describe the rate of change in a function graph by quantifying covariation between two variables (e.g., as *x* increases by 2 units, *y* decreases by 3 units). | of linear functions to interpret the meaning of a point on a linear function graph [e.g., the point (5, 2) might represent that 5 people need 2 medium-sized pizzas]. | |

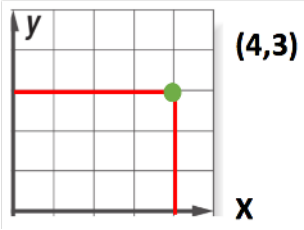
## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to analyze function graphs, students begin by learning to notice what is new. The educator draws the students attention to new objects or stimuli, labels them (e.g., “this set has all red objects; this set has all blue”, “these fidgets are big; these fidgets are small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different. Educators provide sorting activities that allow learners to isolate specific attributes while recognizing likenesses and differences among objects. Educators also provide activities that reinforce the skill of ordering (e.g., arrangement of objects from largest to smallest, sequencing daily events, and counting).

***How is the Distal Precursor related to the Target?***

As students' attention to objects and details develops, educators can extend their attention by providing experience with finding and creating simple patterns using objects and moving to symbols (e.g., numerals). Educators should take care to start with simple patterns (e.g., 1-2-1-2) and take advantage of the symbols that are already being used in the classroom. Educators should demonstrate how students can create and identify the pattern/rule (e.g., using colored cubes, the student creates a line of 5 cubes; the educator then creates a matching set and explains what to do to follow the student's pattern. Then, the student generates a third matching set. If the order is not followed, it is a good teaching opportunity to talk about why it doesn't fit the pattern). Learning to identify the rule of patterns will help students extend their thinking across patterns. As students are working on identifying pattern rules, educators can also begin to demonstrate how rules can be used with ordered pairs. Provide students lots of opportunities to apply rules to create their own examples of ordered pairs. Educators should demonstrate how students can use their counting skills to figure out where to mark the point by counting how far along and how far up the *x*- and *y*-axes.

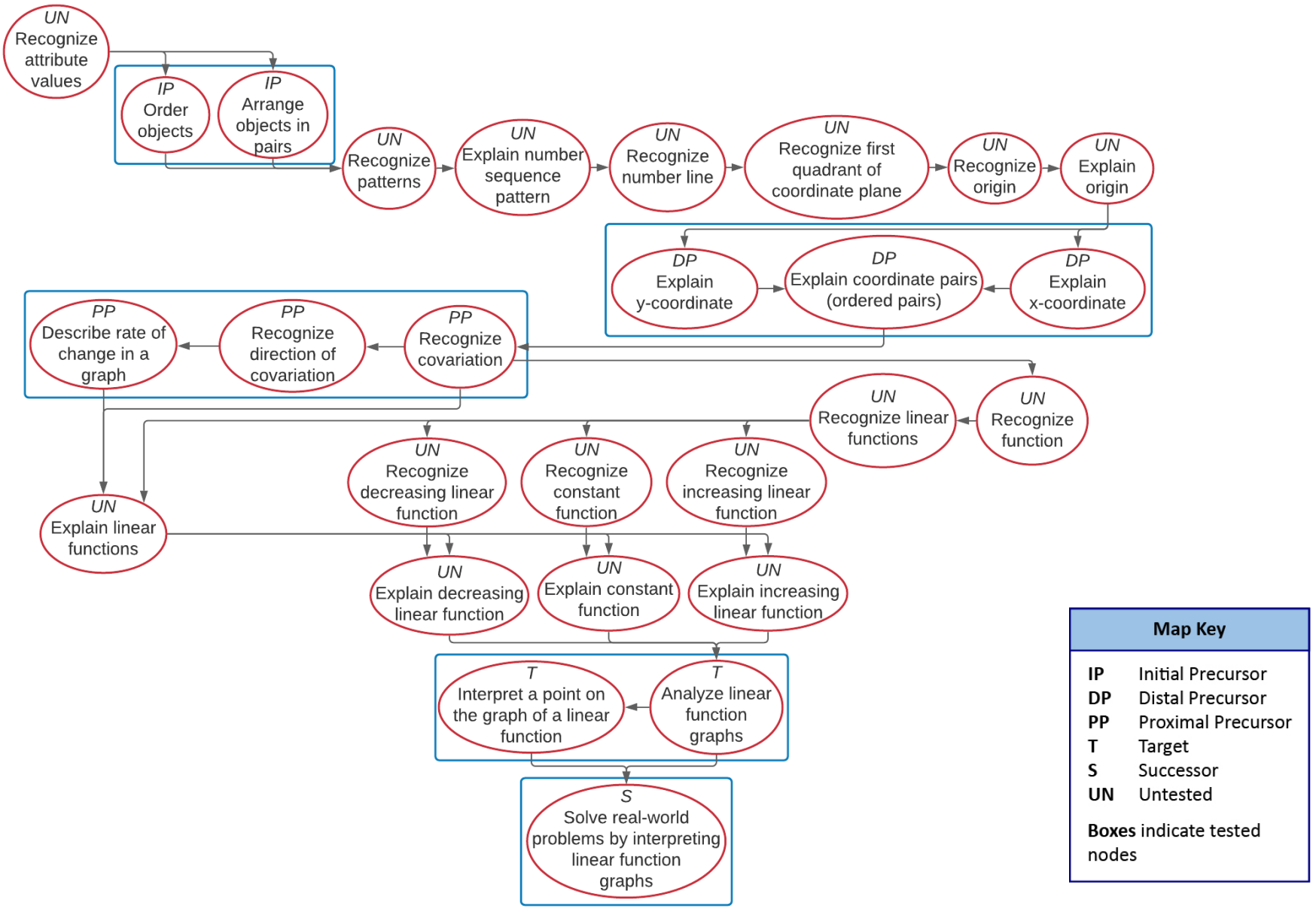


## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

**M.EE.HS.A.REI.10-12** Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.



# Mini-Map for M.EE.HS.A.SSE.4

Subject: Mathematics
Algebra—Seeing Structure in Expressions (A.SSE)
Grade: 11

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.A.SSE.4** Determine the successive term in a geometric sequence given the common ratio. | **M.A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Group together objects by attribute values such as shape or size (e.g., group together a square, a rectangle, and a rhombus, as they all have four sides). Contrast or distinguish objects based on attributes, such as shape, size, texture, and numerical pattern. Order objects by following a specific rule (e.g., arrange three objects with different sizes from the smallest to largest). | Recognize patterns (i.e., repeating, growing, shrinking) involving numbers or letters (e.g., a, b, b, a, b, b...; 2, 5, 8, 11...). Identify a sequence as an ordered list of numbers that adheres to a common rule between corresponding numbers (e.g., 2, 4, 6, 8...). | Recognize a geometric sequence as an ordered list of numbers, such that each term after the first is determined by multiplying or dividing the preceding term by a constant amount (e.g., 2, 4, 8, 16...). Recognize the recursive rule in geometric sequences by determining how each term in the sequence differs from the preceding term (e.g., the recursive rule in the sequence 2, 4, 8, 16... is "multiply by 2"). | Communicate the next term in a geometric sequence by determining how each term in a sequence is obtained from the previous term (e.g., the next term in the geometric sequence 2, 4, 8, 16... is 32). | Determine any term in a geometric sequence when the first term, common ratio, and the nth term formula of a geometric sequence are given [e.g., given the $n$th term formula, $a_n = ar^{(n-1)}$, first term as 2 and the common ratio 3, the 4th term will be $2 \times 3^{(4-1)} = 2 \times 3^3 = 54$]. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to determine a geometric sequence (e.g., 2, 4, 8, 16, 32), students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “there are two cubes”, “this is a circle”, “this fidget is big and this fidget is small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different.

***How is the Distal Precursor related to the Target?***

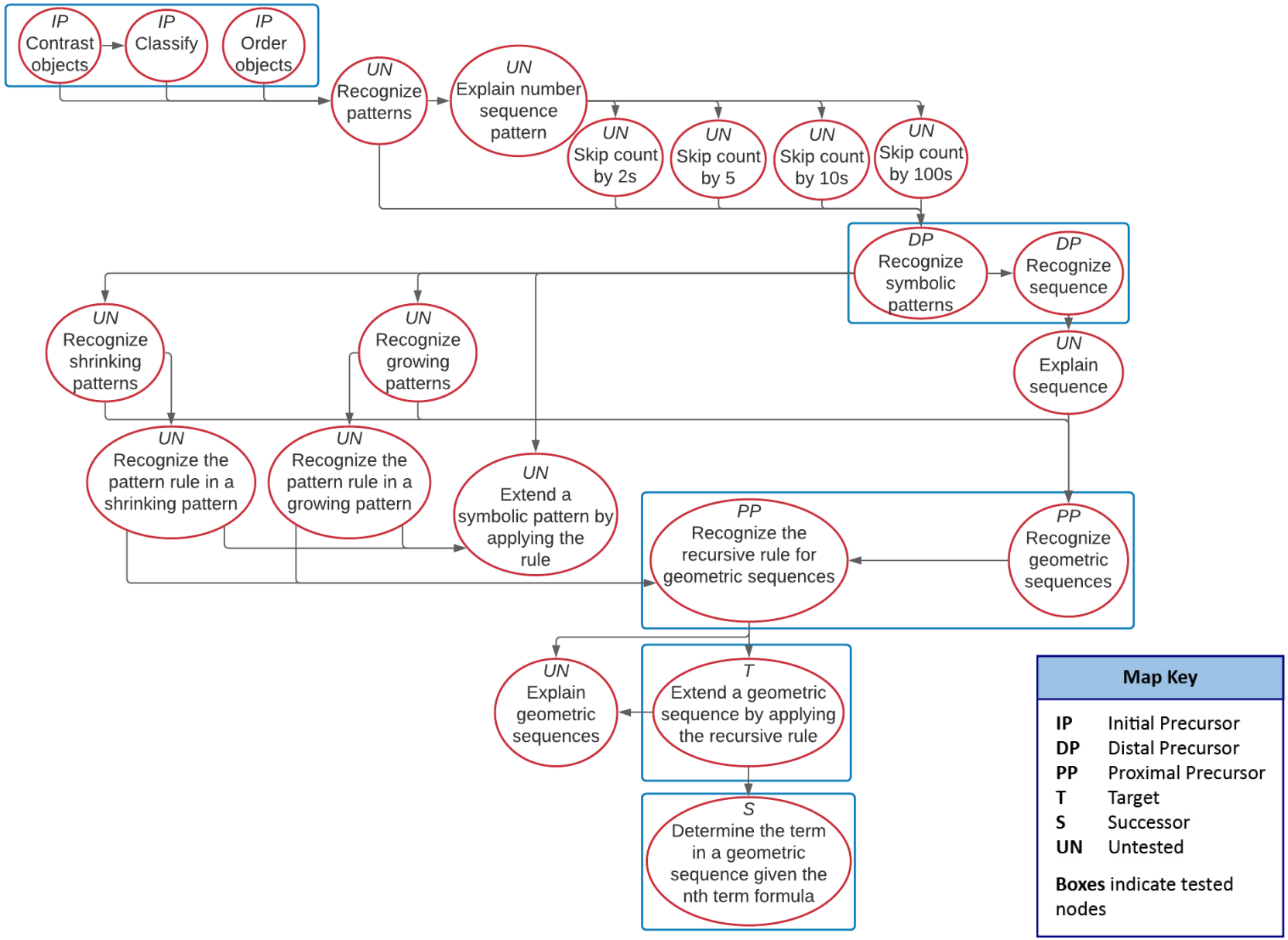
As students develop their understanding of attributes and work toward geometric sequences, educators provide interactive lessons around patterns using attributes like shape, size, and color. At this level, students are also expected to recognize symbolic (e.g., number) patterns. This also requires that students recognize numerals in order (i.e., 1, 2, 3...). Educators should take care to use number names while defining and demonstrating symbolic sequences. While students do not need to say these words, they do need to learn the meanings and the sequence.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

**M.EE.HS.A.SSE.4** Determine the successive term in a geometric sequence given the common ratio.



# Mini-Map for M.EE.HS.F.BF.1

Subject: Mathematics
Functions—Building Functions (F.BF)
Grade: 10

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.F.BF.1** Select the appropriate graphical representation (first quadrant) given a situation involving constant rate of change. | **M.F.BF.1** Write a function that describes a relationship between two quantities. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Arrange objects in a specific order (e.g., smallest to largest). Form a pair by putting together two objects (e.g., putting together a pencil and a ruler). | Communicate understanding that a coordinate pair (ordered pair) is a set of numbers used to show a position on a graph. The first number, "*x*," or the *x*-coordinate in the coordinate pair (*x*, *y*), represents *x* units left or right on the *x*-axis. The second number, "*y*," or the *y*-coordinate, represents *y* units up or down on the *y*-axis [e.g, (4, 8) represents 4 units right on the *x*-axis and 8 units up on the *y*-axis]. | Recognize covariation as the pattern in which two variables or quantities change together. Recognize the direction in which two variables change together (e.g., as *x* increases, *y* decreases). Describe the rate of change in a function graph by quantifying covariation between two variables (e.g., as *x* increases by 2 units, *y* decreases by 3 units). | Represent a real-world problem in the form of a graph. | Find solutions to real-world problems by interpreting linear function graphs. |

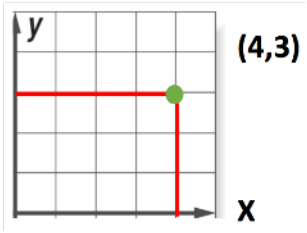
## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to represent real-world problems on graphs, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “this set has all red objects; this set has all blue”, “these fidgets are big; these fidgets are small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different. Educators provide sorting activities that allow learners to isolate specific attributes while recognizing likenesses and differences among objects. Educators also provide activities that reinforce the skill of ordering (e.g., arrangement of objects from largest to smallest, sequencing daily events, and counting).

***How is the Distal Precursor related to the Target?***

As students' attention to objects and details develops, educators can extend their attention by providing experience with finding and creating simple patterns using objects and moving to symbols (e.g., numerals). Educators should take care to start with simple patterns (e.g., 1-2-1-2) and take advantage of the symbols that are already being used in the classroom. Educators should demonstrate how students can create and identify the pattern/rule (e.g., using colored cubes, the student creates a line of 5 cubes; the educator then creates a matching set and explains what to do to follow the student's pattern. Then, the student generates a third matching set. If the order is not followed, it is a good teaching opportunity to talk about why it doesn't fit the pattern). Learning to identify the rule of patterns will help students extend their thinking across patterns. As students are working on identifying pattern rules, educators can also begin to demonstrate how rules can be used with ordered pairs. Provide students lots of opportunities to apply rules to create their own examples of ordered pairs. Educators should demonstrate how students can use their counting skills to figure out where to mark the point by counting how far along and how far up the *x*- and *y*-axes.

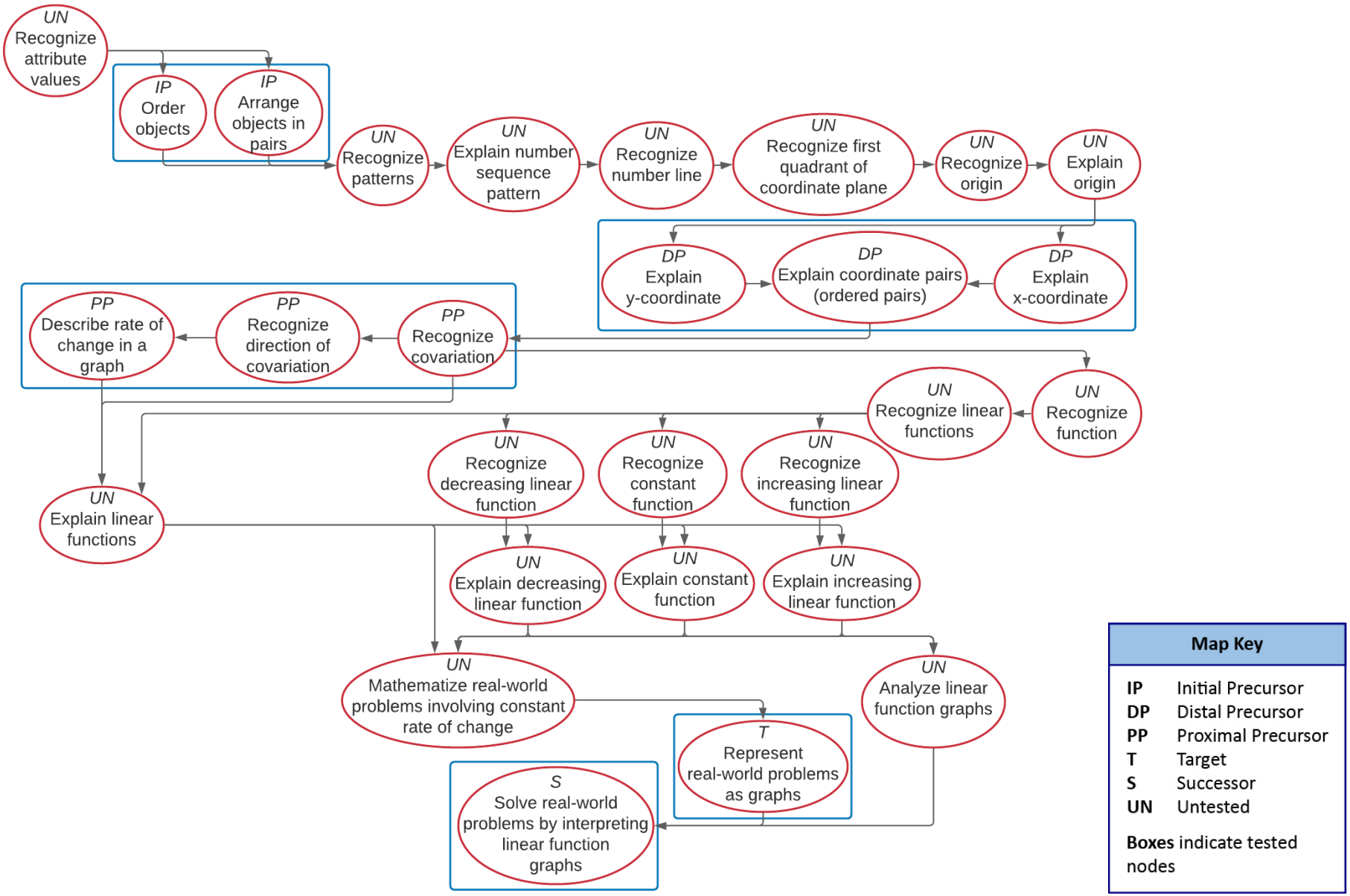


## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

**M.EE.HS.F.BF.1** Select the appropriate graphical representation (first quadrant) given a situation involving constant rate of change.



# Mini-Map for M.EE.HS.F.BF.2

Subject: Mathematics
Functions—Building Functions (F.BF)
Grade: 11

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.F.BF.2** Determine an arithmetic sequence with whole numbers when provided a recursive rule. | **M.F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Group together objects by attribute values such as shape or size (e.g., group together a square, a rectangle, and a rhombus, as they all have four sides). Contrast or distinguish objects based on attributes such as shape, size, texture, and numerical pattern. Order objects by following a specific rule (e.g., arrange three objects with different sizes from the smallest to largest). | Recognize patterns (i.e., repeating, growing, shrinking) involving numbers or letters (e.g., a, b, b, a, b, b...; 2, 5, 8, 11...). Identify a sequence as an ordered list of numbers that adheres to a common rule between corresponding numbers (e.g., 2, 4, 6, 8...). | Recognize an arithmetic sequence as an ordered list of numbers, such that each term after the first is determined by adding or subtracting the preceding term by a constant amount (e.g., 2, 4, 8, 16...). Recognize the recursive rule in arithmetic sequences by determining how each term in the sequence differs from the preceding term (e.g., the recursive rule in the sequence 2, 4, 6, 8... is "add 2"). | Communicate the next term in an arithmetic sequence by determining how each term in a sequence is obtained from the previous term (e.g., the next term in the sequence 2, 4, 6, 8... is 10). | Determine any term in an arithmetic sequence when the first term ($a$), common difference ($d$), and the $n$th term formula for an arithmetic sequence are given [e.g., when the $n$th term formula is $a + d(n - 1)$, the first term is 5, and the common difference is 3, the 6th term equals $5 + 3(6 - 1) = 20$]. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to determine an arithmetic sequence (e.g., 1, 4, 7, 10, 13), students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “there are two cubes”, “this is a circle”, “this fidget is big and this fidget is small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different.

***How is the Distal Precursor related to the Target?***

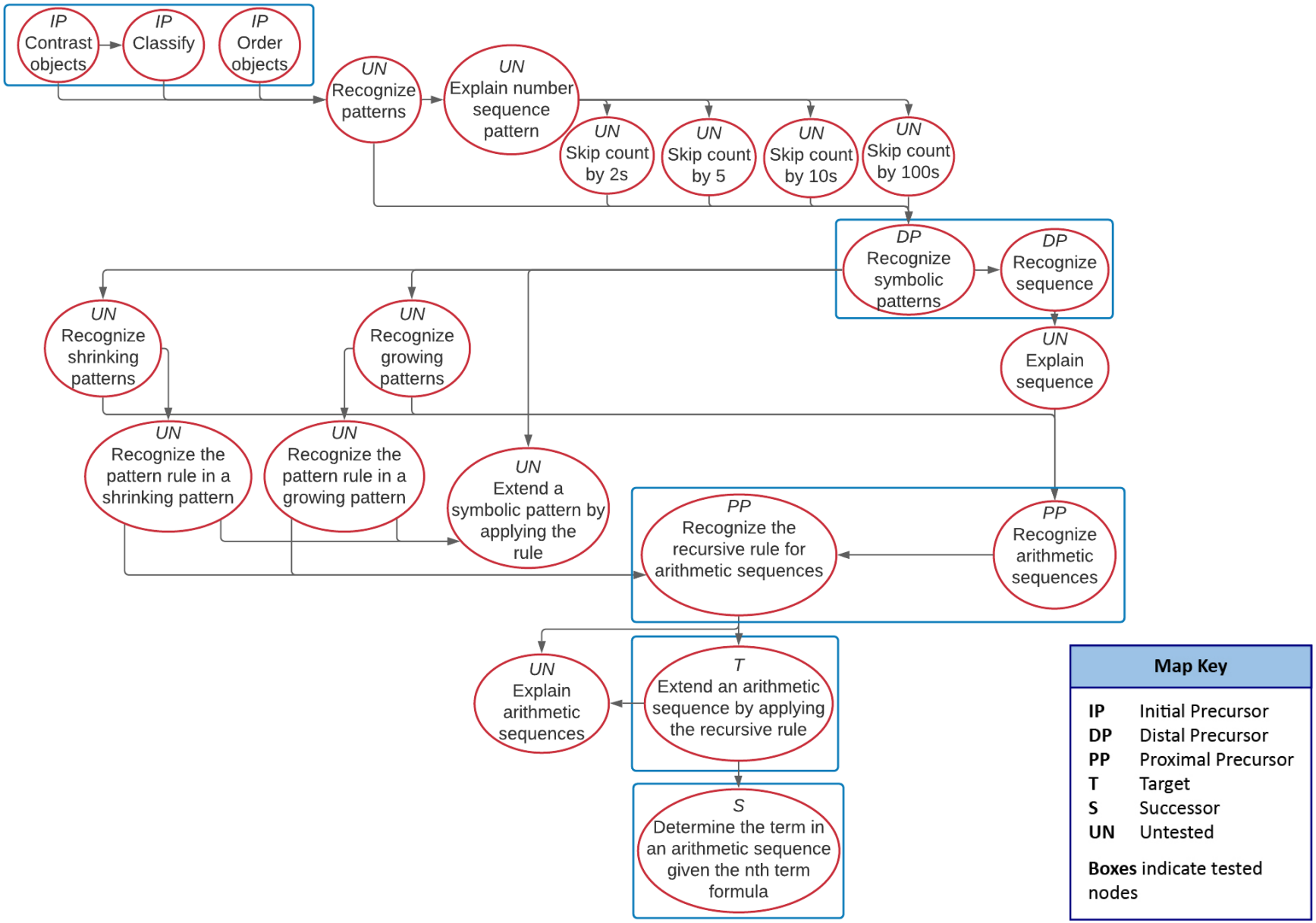
As students develop their understanding of attributes and work towards arithmetic sequences, educators provide interactive lessons around patterns using attributes like shape, size, and color. At this level, students are also expected to recognize symbolic (e.g. numbers) patterns. This also requires that students recognize numerals in order (i.e., 1, 2, 3...). Educators should take care to use number names while defining and demonstrating symbolic sequences. While students do not need to say these words, they do need to learn the meanings and the sequence.

## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

**M.EE.HS.F.BF.2** Determine an arithmetic sequence with whole numbers when provided a recursive rule.



# Mini-Map for M.EE.HS.F.IF.1-3

Subject: Mathematics
Functions—Interpreting Functions (F.IF)
Grade: 11

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.F.IF.1-3** Use the concept of function to solve problems. | **M.F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.<br>**M.F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.<br>**M.F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n – 1)$ for $n \geq 1$. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Arrange objects in a specific order (e.g., smallest to largest). Form a pair by putting together two objects (e.g., putting together a pencil and a ruler). | Communicate understanding that a coordinate pair (ordered pair) is a set of numbers used to show a position on a graph. The first number, "$x$," or the $x$-coordinate in the coordinate pair ($x$, $y$), represents $x$ units left | Describe the rate of change in a function graph and a function table by quantifying the covariation between two variables (e.g., describes that as $x$ increases by 2 units, $y$ decreases by 3 units). | Find solutions to real-world problems by interpreting linear function graphs and tables. | Extend, predict, or infer information presented in linear function graphs and function tables. |

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| | or right on the *x*-axis. The second number, "*y*," or the *y*-coordinate, represents y units up or down on the *y*-axis [e.g., (4, 8) represents 4 units right on the *x*-axis and 8 units up on the *y*-axis]. | | | |

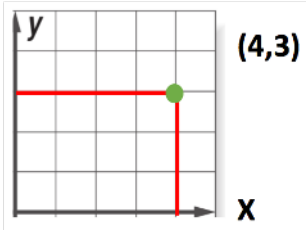
## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to use functions to solve problems, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “this set has all red objects; this set has all blue”, “these fidgets are big; these fidgets are small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different. Educators provide sorting activities that allow learners to isolate specific attributes while recognizing likenesses and differences among objects. Educators also provide activities that reinforce the skill of ordering (e.g., arrangement of objects from largest to smallest, sequencing daily events, and counting).

***How is the Distal Precursor related to the Target?***

As students' attention to objects and details develops, educators can extend their attention by providing experience with finding and creating simple patterns using objects and moving to symbols (e.g., numerals). Educators should take care to start with simple patterns (e.g., 1-2-1-2) and take advantage of the symbols that are already being used in the classroom. Educators should demonstrate how students can create and identify the pattern/rule (e.g., using colored cubes, the student creates a line of 5 cubes; the educator then creates a matching set and explains what to do to follow the student's pattern. Then, the student generates a third matching set. If the order is not followed, it is a good teaching opportunity to talk about why it doesn't fit the pattern). Learning to identify the rule of patterns will help students extend their thinking across patterns. As students are working on identifying pattern rules, educators can also begin to demonstrate how rules can be used with ordered pairs. Provide students lots of opportunities to apply rules to create their own examples of ordered pairs. Educators should demonstrate how students can use their counting skills to figure out where to mark the point by counting how far along and how far up the *x*- and *y*-axes.

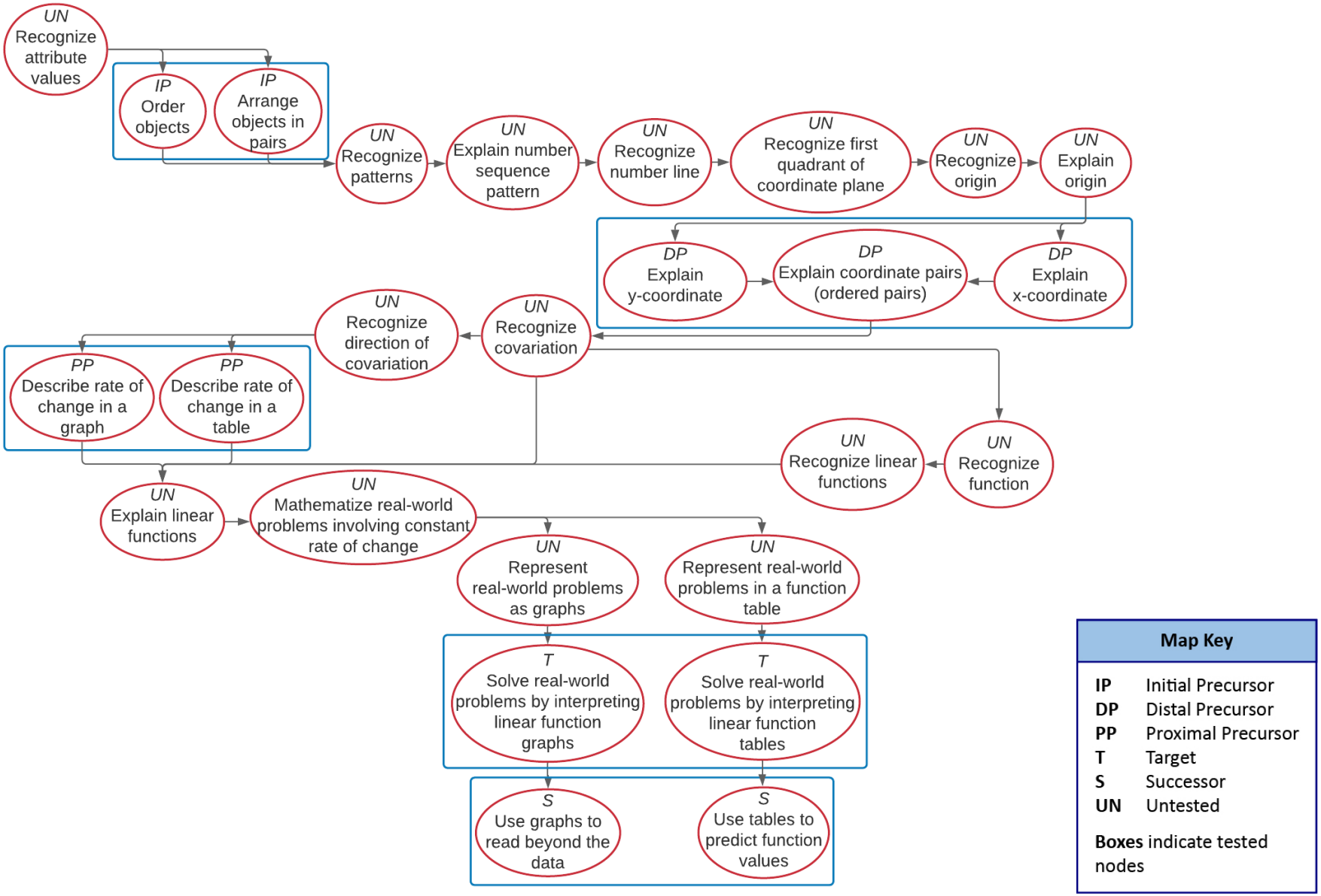


## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

**M.EE.HS.F.IF.1-3** Use the concept of function to solve problems.



# Mini-Map for M.EE.HS.F.IF.4-6

Subject: Mathematics
Functions—Interpreting Functions (F.IF)
Grade: 11

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.F.IF.4-6** Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc. | **M.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.<br>**M.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.<br>**M.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Arrange objects in a specific order (e.g., smallest to largest). Form a pair by putting together two objects (e.g., putting together a pencil and a ruler). | Communicate understanding that a coordinate pair (ordered pair) is a set of numbers used to show a position on a graph. The first number, "*x*," or | Recognize covariation as the pattern in which two variables or quantities change together. Recognize the direction in which two variables change | Communicate whether a linear function graph has an increasing, decreasing, or constant rate of change. Compare two functions with different rates of | Solve real-world problems by interpreting linear function graphs. Compare rates of change, *x*- and *y*-intercepts, direction of |

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| | the x-coordinate in the coordinate pair (*x*, *y*), represents *x* units left or right on the *x*-axis. The second number, "*y*," or the *y*-coordinate, represents *y* units up or down on the *y*-axis [e.g., (4, 8) represents 4 units right on the *x*-axis and 8 units up on the *y*-axis]. | together (e.g., as *x* increases, *y* decreases). Describe the rate of change in a function graph by quantifying covariation between two variables (e.g., as *x* increases by 2 units, *y* decreases by 3 units). | change to communicate which function is faster or slower, higher or lower. | change (covariation), or overall shape of two linear functions represented graphically. |

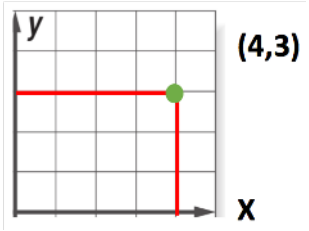
## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to construct graphs that represent a linear function, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “this set has all red objects; this set has all blue”, “these fidgets are big; these fidgets are small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different. Educators provide sorting activities that allow learners to isolate specific attributes while recognizing likenesses and differences among objects. Educators also provide activities that reinforce the skill of ordering (e.g., arrangement of objects from largest to smallest, sequencing daily events, and counting).

***How is the Distal Precursor related to the Target?***

As students' attention to objects and details develops, educators can extend their attention by providing experience with finding and creating simple patterns using objects and moving to symbols (e.g., numerals). Educators should take care to start with simple patterns (e.g., 1-2-1-2) and take advantage of the symbols that are already being used in the classroom. Educators should demonstrate how students can create and identify the pattern/rule (e.g., using colored cubes, the student creates a line of 5 cubes; the educator then creates a matching set and explains what to do to follow the student's pattern. Then, the student generates a third matching set. If the order is not followed, it is a good teaching opportunity to talk about why it doesn't fit the pattern). Learning to identify the rule of patterns will help students extend their thinking across patterns. As students are working on identifying pattern rules, educators can also begin to demonstrate how rules can be used with ordered pairs. Provide students lots of opportunities to apply rules to create their own examples of ordered pairs. Educators should demonstrate how students can use their counting skills to figure out where to mark the point by counting how far along and how far up the *x*- and *y*-axes.

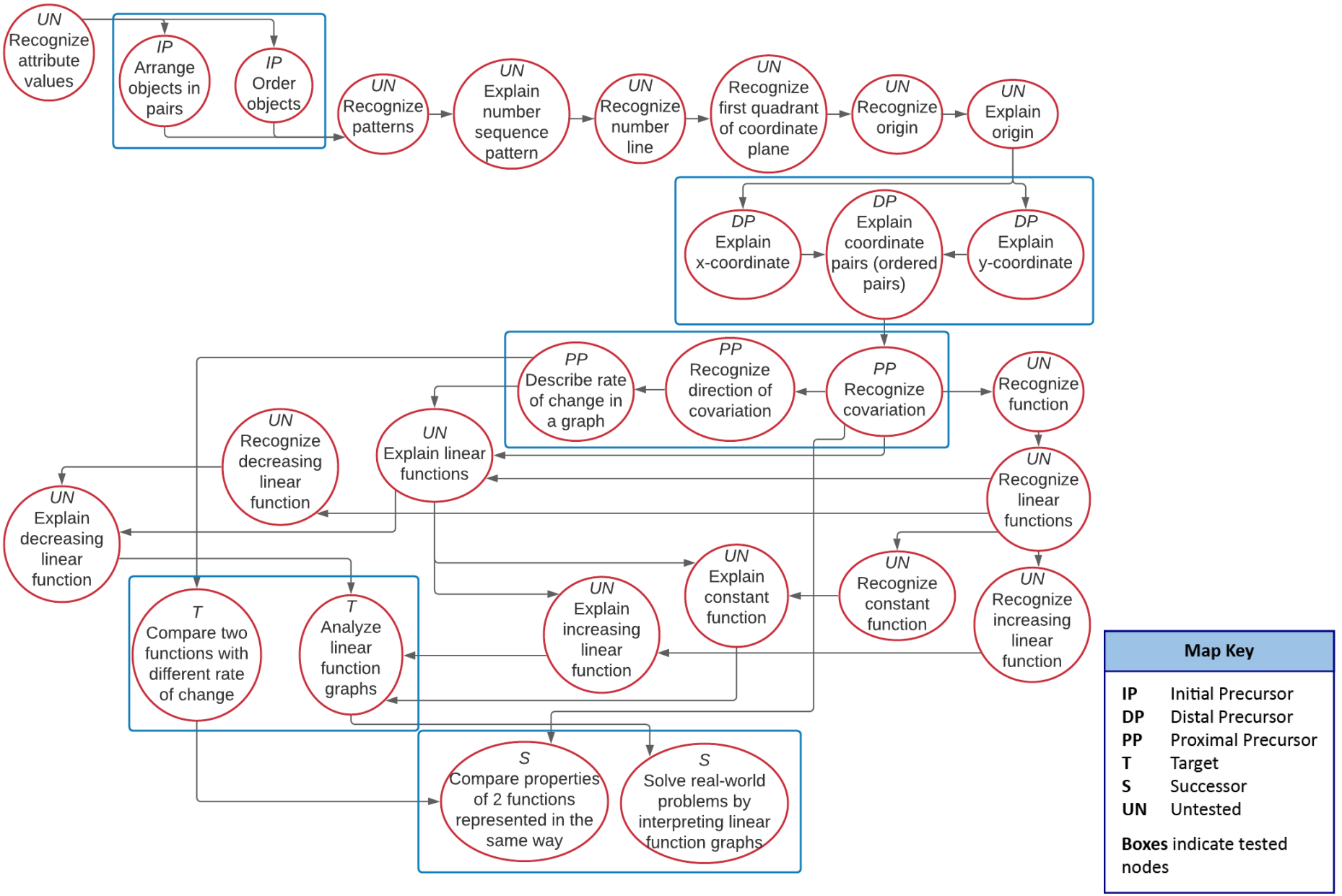


## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

**M.EE.HS.F.IF.4-6** Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc.



# Mini-Map for M.EE.HS.F.LE.1-3

Subject: Mathematics
Functions—Linear, Quadratic, and Exponential Models (F.LE)
Grade: 11

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
|---|---|
| **M.EE.HS.F.LE.1-3** Model a simple linear function such as $y = mx$ to show that these functions increase by equal amounts over equal intervals. | **M.F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.<br>**M.F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).<br>**M.F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| Arrange objects in a specific order (e.g., smallest to largest).<br>Form a pair by putting together two objects (e.g., putting together a pencil and a ruler). | Communicate understanding that a coordinate pair (ordered pair) is a set of numbers used to show a position on a graph.<br>The first number, "*x*," or the *x*-coordinate in the coordinate pair (*x*, *y*), represents *x* units left or right on the *x*-axis.<br>The second number, "*y*," or the *y*-coordinate, | Recognize covariation as the pattern in which two variables or quantities change together. Recognize the direction in which two variables change together (e.g., as *x* increases, y decreases).<br>Calculate slope given sets of coordinate pairs [e.g., given a set of coordinates (4, 5) and | Determine the rate of change of linear functions.<br>Communicate understanding that the average rate of change is the ratio between the change in a quantity over an interval of time. | Identify or name intervals where the function is increasing or decreasing. Estimate average rate of change when given a graph. |

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
|---|---|---|---|---|
| | represents *y* units up or down on the *y*-axis [e.g., (4, 8) represents 4 units right on the *x*-axis and 8 units up on the *y*-axis]. | (6, 10), calculate slope as (10 - 5)/(6 - 4) = 5/2]. | | |

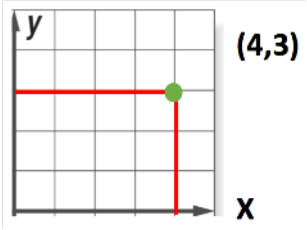
## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

***How is the Initial Precursor related to the Target?***

In order to model linear functions, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., “this set has all red objects; this set has all blue”, “these fidgets are big; these fidgets are small”), and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different. Educators provide sorting activities that allow learners to isolate specific attributes while recognizing likenesses and differences among objects. Educators also provide activities that reinforce the skill of ordering (e.g., arrangement of objects from largest to smallest, sequencing daily events, and counting).

***How is the Distal Precursor related to the Target?***

As students' attention to objects and details develops, educators can extend their attention by providing experience with finding and creating simple patterns using objects and moving to symbols (e.g., numerals). Educators should take care to start with simple patterns (e.g., 1-2-1-2) and take advantage of the symbols that are already being used in the classroom. Educators should demonstrate how students can create and identify the pattern/rule (e.g., using colored cubes, the student creates a line of 5 cubes; the educator then creates a matching set and explains what to do to follow the student's pattern. Then, the student generates a third matching set. If the order is not followed, it is a good teaching opportunity to talk about why it doesn't fit the pattern). Learning to identify the rule of patterns will help students extend their thinking across patterns. As students are working on identifying pattern rules, educators can also begin to demonstrate how rules can be used with ordered pairs. Provide students lots of opportunities to apply rules to create their own examples of ordered pairs. Educators should demonstrate how students can use their counting skills to figure out where to mark the point by counting how far along and how far up the *x*- and *y*-axes.



## Instructional Resources

| Released Testlets |
| --- |
| See the Guide to Practice Activities and Released Testlets. |
| **Using Untested (UN) Nodes** |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

**M.EE.HS.F.LE.1-3** Model a simple linear function such as y = mx to show that these functions increase by equal amounts over equal intervals.