# Mini-Map for M.EE.8.NS.2.a 

LEARNING MAPS
Subject: Mathematics
The Number System (NS)
Grade: 8

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
| :--- | :--- |
| M.EE.8.NS.2.a Express a fraction with a denominator of 100 as <br> a decimal. | M.8.NS.2 Use rational approximations of irrational numbers to <br> compare the size of irrational numbers, locate them <br> approximately on a number line diagram, and estimate the <br> value of expressions (e.g., $\left.\pi^{2}\right)$. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Communicate understanding of "separateness" by recognizing objects that are not joined together. Communicate understanding of a set by recognizing a group of objects sharing an attribute. | Divide a set (e.g., 8 crayons) into two or more equal subsets (e.g., two subsets of 4 crayons). Demonstrate understanding of a unit fraction (e.g., 1/4) as the quantity formed by one part when a whole is partitioned into $n$ (e.g., 4) equal parts. | Communicate understanding that a decimal point is a dot that separates the whole number from the fractional part of a number. Represent a fraction with a denominator of 10 as a decimal. | Represent a fraction with a denominator of 100 as a decimal (e.g., $52 / 100$ as 0.52 ). | Compare two decimals to the tenths or hundredths place using symbols (i.e., =, <, >) to show that one is greater than, less than, or equal to the other. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

How is the Initial Precursor related to the Target?
Converting a fraction to a decimal requires a student to be able to recognize that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. As educators present a set, they label it (e.g., two balls, one marker, three CDs), count the items, label it again, and encourage students to use numerals to label and count the separate sets. Use tools like the ten-frame to point out whole and parts (e.g., a row of 5 dots and a row of 4 dots are parts or subsets of 9 ).

## 000

Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

## How is the Distal Precursor related to the Target?

As students become more adept at tracking discrete objects, they will begin working on one-to-one distribution of objects to person, objects to objects, and objects to available space (e.g., giving each person in the group a pencil; given four counters, they would line up four more counters in front of or on top of the first set; given three chairs at a table, the student would place a cup on the table for each available chair). As students understanding of one-to-one distribution develops, provide students many opportunities to recognize equivalence in sets with same items and then sets with differing items. As students work on all these skills and concepts, continue to draw their attention to parts and wholes.

## Link to Text-Only Map

M.EE.8.NS.2.a Express a fraction with a denominator of 100 as a decimal.


# Mini-Map for M.EE.8.NS.2.b 

LEARNING MAPS
Subject: Mathematics
The Number System (NS)
Grade: 8

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
| :--- | :--- |
| M.EE.8.NS.2.b Compare quantities represented as decimals in <br> real-world examples to hundredths. | M.8.NS.2 Use rational approximations of irrational numbers to <br> compare the size of irrational numbers, locate them <br> approximately on a number line diagram, and estimate the <br> value of expressions (e.g., $\pi^{2}$ ). |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Recognize separateness as objects that are not joined together. | Recognize a set model that represents onetenth or multiple tenths, such as twotenths, five-tenths, or eight-tenths. | Represent a decimal to tenths (e.g., 5.2) or hundredths (e.g., 7.68) as a fraction (i.e., 52/10 and 768/100, respectively). | Compare two decimals to the hundredths place using symbols (i.e., =, <, $>$ ) to show that one is greater than, less than, or equal to the other. | Compare two decimals to the thousandths (e.g., 1.050 and 1.762) using symbols (i.e., $=$, , , $>$ ) to show that one is greater than, less than, or equal to the other (e.g., $1.050<1.762$ ). |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

## How is the Initial Precursor related to the Target?

Representing fractions as decimals requires a student to be able to recognize that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. As educators present a set, they label it (e.g., two balls, one marker, three CDs), count the items, label it again, and encourage students to use numerals to label and count the separate sets. Use tools like the ten-frame to point out whole and parts (e.g., a row of 5 dots and a row of 4 dots are parts or subsets of 9 ).

## 0000

## Instructional Resources

## Released Testlets

See the Guide to Practice Activities and Released Testlets. Using Untested (UN) Nodes

See the document Using Mini-Maps to Plan Instruction.

## How is the Distal Precursor related to the Target?

As students begin to understand labeling, counting small sets, and recognizing wholes and parts of objects and sets, use set models to provide a wide variety of sets of 10 to model tenths (e.g., for individual shapes to match the fraction, say, "I have 10 cubes in my bag, 1/10 of them are blue.").

## Link to Text-Only Map

M.EE.8.NS.2.b Compare quantities represented as decimals in real-world examples to hundredths.


Map Key
IP Initial Precursor
DP Distal Precursor
PP Proximal Precursor
T Target
S Successor
UN Untested
Boxes indicate tested nodes

# Mini-Map for M.EE.8.EE. 1 

learning maps

## Subject: Mathematics <br> Expressions and Equations (EE) <br> Grade: 8

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
| :--- | :--- |
| M.EE.8.EE.1 Identify the meaning of an exponent (limited to <br> exponents of 2 and 3). | M.8.EE.1 Know and apply the properties of integer exponents <br> to generate equivalent numerical expressions. |

Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Combine two or more sets of objects to create a new set. Combine two or more parts (e.g., toys, shapes) to form a new whole. <br> Demonstrate an understanding of addition by combining the objects of two or more sets. | Communicate understanding that in repeated addition problems, a single numerical value is added repeatedly (e.g., $6+6+6$ ) and that one way to add a number a given number of times is by using skip-counting as a strategy (e.g., $6+6$ +6 can be added as 6 , $12,18)$. Use models, such as mathematical equations (e.g., $5+5+$ $5=15$ ), sets of manipulatives, or number line diagrams to represent a repeated addition problem. | Demonstrate multiplication by combining multiple sets containing the same number of objects. Communicate understanding that the number of sets times the number of objects in each set equals the total number of objects. Communicate understanding that in multiplication, one factor represents the number of elements in a group, the second factor represents the number of groups, and the product is the | Recognize exponents [i.e., "b", in expressions $a^{b}$, where " $b$ " indicates the number of times the base number ("a") is to be multiplied (e.g., $\left.\left.2^{3}=2 \times 2 \times 2\right)\right]$. | Explain that when multiplying two base numbers raised to the same power, the problem equals the product of the base numbers with the same exponent, and that when multiplying (or dividing) two base numbers raised to different powers, the problem equals the product (or quotient) of the base numbers raised to the sum (or difference) of the exponents. Solve for when a nonzero number is raised to the |

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| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
|  |  | number obtained by <br> multiplying two factors. | 0 power, where the <br> answer is always one. |  |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

How is the Initial Precursor related to the Target?
Recognizing exponents requires a student to be able to recognize that two or more sets or groups of items exist. Educators can work on this skill using a variety of sets. Help students recognize when items are grouped together into a set and when they are separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numbers to label and count the separate sets. Then, combine the sets, give it a new label, and count the set.

## How is the Distal Precursor related to the Target?

As students' understanding of labeling and counting sets develops, they will begin working on adding items to a set and combining sets to create a new set. Additionally, students will work on developing an understanding of equal shares by actively participating in one-to-one distribution of objects to person, objects to objects, and objects to available space (e.g., giving each person in the group a pencil; given four counters, they would line up four more counters in front of or on top of the first set; given three chairs at a table, the student would place a cup on the table for each available chair). As students learn to work with sets and connect their understanding of equal shares to sets, educators can provide students experience with combining multiple sets (e.g., 3 sets with 4 counters each) and represent the problem (e.g., $4+4+4=$ ?). Students will also learn to represent the problem using a pencil or their communication system (e.g., the student is shown 4 equal sets each with 2 counters. The student counts the first set and writes a 2 or indicates 2 , then writes or indicates the plus sign. The student repeats for all 4 sets and then indicates the equal sign and solves the problem.).

## Instructional Resources

Released Testlets
See the Guide to Practice Activities and Released Testlets. Using Untested (UN) Nodes

See the document Using Mini-Maps to Plan Instruction.

## Link to Text-Only Map

M.EE.8.EE. 1 Identify the meaning of an exponent (limited to exponents of 2 and 3).


# Mini-Map for M.EE.8.NS. 1 

LEARNING MAPS
Subject: Mathematics
The Number System (NS)
Grade: 8

## Learning Outcome

## DLM Essential Element

M.EE.8.NS. 1 Subtract fractions with like denominators (halves, thirds, fourths, and tenths) with minuends less than or equal to one.

## Grade-Level Standard

M.8.NS. 1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Communicate understanding of "separateness" by recognizing objects that are not joined together. Communicate understanding of a subset by recognizing a subset as a set or group of objects within a larger set that share an attribute. | Recognize each object as the part of a whole or unit when shown a whole or unit containing a group of objects. | Communicate understanding that when fractional parts are added, it produces a larger portion of the whole, and that when fractional parts are separated, it results in a smaller portion of the whole. Decompose fractions into sums of unit fractions with the same denominator (e.g., $3 / 7=1 / 7+1 / 7+$ 1/7). | Subtract two fractions with common denominators (e.g., 4/5 $-1 / 5=3 / 5)$. | $\begin{aligned} & \text { Add or subtract two } \\ & \text { fractions where one } \\ & \text { fraction has a } \\ & \text { denominator of } 10 \text { and } \\ & \text { one has a denominator } \\ & \text { of } 100 \text { (e.g., } 5 / 10+ \\ & 1 / 100=50 / 100+1 / 100 \\ & =51 / 100 \text { ). } \end{aligned}$ |

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## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

## How is the Initial Precursor related to the Target?

Subtracting fractions requires a student to be able to recognize that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. As educators present a set, they label it (e.g., two balls, one marker, three CDs), count the items, label it again, and encourage students to use numerals to label and count the separate sets. Use tools like the ten-frame to point out whole and parts (e.g., a set of 9 is part of 10).

How is the Distal Precursor related to the Target? As students work toward greater understanding of sets, educators will provide students with many set models (see below) of fractions using the same unit fraction, either halves, thirds, fourths, or tenths. Students will work on identifying the whole.

Unit Fraction 1/2


Unit Fraction $1 / 4$


|  |  |  |  |
| :--- | :--- | :--- | :--- |

## Unit Fraction 1/3



Unit Fraction 1/10


## Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

M.EE.8.NS. 1 Subtract fractions with like denominators (halves, thirds, fourths, and tenths) with minuends less than or equal to one.


## Mini-Map for M.EE.8.G. 1

LEARNING MAPS

## Subject: Mathematics

Geometry (G)
Grade: 8

## Learning Outcome

## DLM Essential Element

M.EE.8.G.1 Recognize translations, rotations, and reflections of shapes.

## Grade-Level Standard

M.8.G. 1 Verify experimentally the properties of rotations, reflections, and translations.

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :--- | :--- | :--- | :--- | :--- |
| Recognize attributes or <br> characteristics of an <br> object, such as color, <br> orientation, length, <br> width, and weight. | Recognize defining <br> attributes (e.g., number <br> of sides, number of <br> angles) versus <br> nondefining attributes <br> of a shape (e.g., color, <br> size, orientation). | Explain that a <br> transformation of a <br> shape (e.g., translation <br> [slide], reflection [flip], <br> rotation [turn]) does <br> not change the size, <br> area, or shape of the <br> figure. | Recognize the figure <br> that is translated from <br> the original view as a <br> translation (slide), <br> reflected from the <br> original view as a <br> reflection (flip), or <br> rotated from the <br> original view as a <br> rotation (turn). | Explain that in <br> transformations (i.e., <br> rotations, reflections, <br> and translations), <br> parallel lines remain <br> parallel, lines remain <br> lines, angle <br> measurements remain <br> constant, and line <br> segments remain line <br> segments of the same <br> length. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

How is the Initial Precursor related to the Target?
Being able to recognize shapes given certain conditions requires a student to recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., this is a square $\quad$ ). Then provide multiple examples of the same shape so students
can make comparisons (e.g., $\square$ ), focusing student attention on the characteristics that make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as "same" or "different."

NOTE: When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size).

## How is the Distal Precursor related to the Target?

Now that students have experience identifying shapes, provide instruction that focuses on the attribute of a given shape and making comparisons with other shapes. Educators should take care to use the names of the shapes while defining and describing the attributes. While students do not need to say the shape names, they do need to learn what makes a shape a shape (e.g., a square has four equal straight sides, a triangle has three straight sides, a cone is an object that narrows from a circular base to a point, and a rectangle does not have curves).

## Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

M.EE.8.G.1 Recognize translations, rotations, and reflections of shapes.


| Map Key |  |
| :--- | :--- |
| IP | Initial Precursor |
| DP | Distal Precursor |
| PP | Proximal Precursor |
| T | Target |
| S | Successor |
| UN | Untested |
| Boxes indicate tested |  |
| nodes |  |

# Mini-Map for M.EE.8.G. 2 

LEARNING MAPS

## Subject: Mathematics

Geometry (G)
Grade: 8

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
| :--- | :--- |
| M.EE.8.G.2 Identify shapes that are congruent. | M.8.G.2 Understand that a two-dimensional figure is congruent <br> to another if the second can be obtained from the first by a <br> sequence of rotations, reflections, and translations; given two <br> congruent figures, describe a sequence that exhibits the <br> congruence between them. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Recognize "same" as the object that shares all of the same attributes as other objects in a group. Recognize "different" as the object that shares some or none of the attributes as other objects in a group. | Match a familiar shape (e.g., square, circle, triangle, rectangle) to a congruent shape (i.e., the shape with same size and orientation), or match a familiar shape (e.g., square, circle, triangle, rectangle) to a similar shape (i.e., the shape shown in a different size but same orientation). | Describe attributes or characteristics of the shape (e.g., size, orientation, the number of sides). Compare shapes and identify attributes shared by the two shapes (e.g., a rectangle and a square each have four sides). | Recognize two shapes that are congruent with or without rotation or reflection. | Communicate understanding that two shapes are congruent if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Describe a sequence of transformations that would result in one figure being superimposed precisely over the other figure. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

How is the Initial Precursor related to the Target?
Being able to recognize congruent figures requires a student to recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., "this is a square" $\quad$ ). Then, provide multiple examples of the same shape so students can make comparisons, focusing student attention on the characteristics make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as same or different.

NOTE: When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size).


## How is the Distal Precursor related to the Target?

As students develop an understanding of same and different shapes, provide opportunities for students to match or group the same shapes based on the shape size (e.g., "this is a big square", "this is a little square"). As students progress with identifying the size of shapes, the educator can begin to introduce different orientations of the shape.

NOTE: As new attributes (e.g., size and orientation) are introduced, be sure to support the student in remembering that the attribute doesn't change the name of the shape.

## Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

## M.EE.8.G.2 Identify shapes that are congruent.



## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
| :--- | :--- |
| M.EE.8.G.4 Identify similar shapes with and without rotation. | M.8.G.4 Understand that a two-dimensional figure is similar to <br> another if the second can be obtained from the first by a <br> sequence of rotations, reflections, translations, and dilations; <br> given two similar two-dimensional figures, describe a sequence <br> that exhibits the similarity between them. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Recognize "same" as the object that shares all of the same attributes as other objects in a group. Recognize "different" as the object that shares some or none of the attributes as other objects in a group. | Match two 2dimensional or 3dimensional shapes (e.g., squares, rectangles, circles, spheres, rectangular prisms, cubes, pyramids) that are different sizes and have the same orientation. | Recognize twodimensional and threedimensional shapes that are similar. Recognize rotation as the transformation in which a shape or figure is turned. | Communicate understanding that two shapes are similar if the second can be obtained from the first by a sequence of dilations, rotations, reflections, or translations. | Describe a sequence of transformations (e.g., dilations, rotations, reflections, translations) that would result in the final shape or figure being similar to the original shape or figure. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

How is the Initial Precursor related to the Target?
Being able to recognize congruent figures requires a student to recognize when basic objects and shapes are the same or different. Work on this understanding by providing students with a shape and naming it (e.g., "this is a square" $\quad$ ). Then, provide multiple examples of the same shape so students can make comparisons, focusing student attention on the characteristics that make this a particular shape (e.g., a square has 4 sides that are the same size). As students explore shapes, label them and describe them as same or different.

NOTE: When presenting the same shape for comparison, do use shapes with different colors, textures, sizes, and orientation so that students understand the attribute that makes it that shape (e.g., 4 sides that are the same size).


## How is the Distal Precursor related to the Target?

As students develop an understanding of same and different shapes, provide opportunities for students to match or group the same two- and three-dimensional shapes based on the shape size (e.g., "this is a big square," "this is a little square"). As students progress with identifying the size of two- and threedimensional shapes, the educator can begin to introduce different orientations of the shape.

NOTE: As new attributes (e.g., size and orientation) are introduced, be sure to support the student in remembering that the attribute doesn't change the name of the shape.

## Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

M.EE.8.G.4 Identify similar shapes with and without rotation.


## Mini-Map for M.EE.8.G.5

LEARNING MAPS

## Subject: Mathematics

Geometry (G)
Grade: 8

## Learning Outcome

## DLM Essential Element

M.EE.8.G.5 Compare any angle to a right angle, and describe the angle as greater than, less than, or congruent to a right angle.

## Grade-Level Standard

M.8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :--- | :--- | :--- | :--- | :--- |
| Recognize attributes or <br> characteristics of an <br> object, such as color, <br> orientation, length, <br> width, and weight. | Recognize an angle as a <br> figure formed by two <br> rays sharing one <br> endpoint. | Recognize angles that <br> are either acute, <br> obtuse, or right. | Compare the measure <br> of an angle to the <br> measure of a right <br> angle, and <br> communicate whether <br> the measure of the <br> angle is greater than, <br> less than, or congruent <br> to the measure of the <br> right angle. | Explain that <br> complementary angles <br> are pairs of angles with <br> measures that add up <br> to 90 degrees (e.g., a <br> 40-degree angle and 50- <br> degree angle). |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

## How is the Initial Precursor related to the Target?

 In order to recognize angles, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., "this is a circle, and it does not have any sides," "this is a rectangle, and it has four sides") and the student observes, feels, or otherwise interacts with the shapes.
## Instructional Resources

| Released Testlets |
| :---: |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

## Released Testlets

See the Guide to Practice Activities and Released Testlets. Using Untested (UN) Nodes

See the document Using Mini-Maps to Plan Instruction.

## How is the Distal Precursor related to the Target?

At this level, educators are providing students with specific vocabulary (line, line segment, point, and ray) that are used to form an angle. These are all denoted by certain characteristics (a line has arrows on both ends; a line segment includes both endpoints; a point is a dot on a graph, a line, line segment, or a number line; a ray is a line that has a well-defined starting point). Educators should take care to use the names "line," "line segment," "point," and "ray" while defining and describing the angles. While students do not need to say the names, they do need to learn their meaning. Educators should teach these attributes within the context of working with angles.

## Link to Text-Only Map

M.EE.8.G.5 Compare any angle to a right angle, and describe the angle as greater than, less than, or congruent to a right angle.


| Map Key |  |
| :--- | :--- |
| IP | Initial Precursor |
| DP | Distal Precursor |
| PP | Proximal Precursor |
| T | Target |
| S | Successor |
| UN | Untested |
| Boxes indicate tested |  |
| nodes |  |

# Mini-Map for M.EE.8.G.9 

LEARNING MAPS

## Subject: Mathematics

Geometry (G)
Grade: 8

## Learning Outcome

## DLM Essential Element

M.EE.8.G.9 Use the formulas for perimeter, area, and volume to solve real-world and mathematical problems (limited to perimeter and area of rectangles and volume of rectangular prisms).

## Grade-Level Standard

M.8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Recognize attributes or characteristics of an object, such as color, orientation, length, width, and weight. | Recognize attributes or characteristics of an object that are measurable (e.g., length, weight, time). | Communicate understanding that length is the distance between the two points that define a line segment, perimeter is the distance that surrounds a plane area, area is the amount of space contained within the outline or boundary of a two-dimensional object or figure, and volume is the space enclosed by a shape or an object. | Calculate area of a rectangle using the area formula (area $=$ length $x$ width), perimeter of a parallelogram using the perimeter formula (perimeter $=2 a+2 b)$, and volume of a prism using the volume formula (volume $=$ height $x$ length $x$ width). | Solve word problems where the unknown quantity is obtained using the volume of a rectangular prism, area of a rectangle, or perimeter of a polygon. |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

How is the Initial Precursor related to the Target?
In order to calculate volume, area, and perimeter with formulas, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., "this is a circle, which has no corners, so we can go all the way around without stopping," "this is a rectangle, which has four corners, two long sides, and two short sides") and the student observes, feels, or otherwise interacts with the shapes. Students also work on counting small units, recognizing that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. As educators present sets, they label them (e.g., two balls, one bear, three blocks), count the items, label them again, and encourage students to use numbers to label and count the separate sets.

## Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

## How is the Distal Precursor related to the Target?

As students develop their attention to objects and notice the difference between objects, they will begin working on recognizing measurable attributes. Students need lots of experience making direct comparisons between objects. Educators should take care to use attribute words like "big"/"small," "tall"/"short," "longer"/"shorter" while defining and demonstrating their meaning. While students do not need to say these words, they do need to learn the meanings.

## Link to Text-Only Map

M.EE.8.G.9 Use the formulas for perimeter, area, and volume to solve real-world and mathematical problems (limited to perimeter and area of rectangles and volume of rectangular prisms).


| Map Key |  |
| :--- | :--- |
| IP | Initial Precursor |
| DP | Distal Precursor |
| PP | Proximal Precursor |
| T | Target |
| S | Successor |
| UN | Untested |
| Boxes indicate tested |  |
| nodes |  |

DLM Essential Element: M.EE.8.G.9
Page 3 of 3

## Learning Outcome

## DLM Essential Element

M.EE.8.SP. 4 Construct a graph or table from given categorical data, and compare data categorized in the graph or table.

## Grade-Level Standard

M.8.SP. 4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Arrange objects in a specific order or by following a specific rule (e.g., arranging three pencils by increasing length). Group like items by attributes and distinguish between like items based on simple characteristics such as shape, size, texture, and numerical pattern. | Recognize the structure of bar graphs, picture graphs, line plots, and tally charts, such as the title and labels for the $x$ - and $y$-axes. <br> Understand that bars are used to display data on bar graphs. <br> Understand that pictures, symbols, or geometric figures are used to display data on picture graphs. <br> Understand that on a line plot, "x" is used to | Answer questions by lifting information from a bar graph, picture graph, line plot, and tally chart and understand the information represented on the graph (e.g., in the graph representing students' favorite ice cream, how many students like strawberry ice cream? How many students like chocolate ice cream?). | Represent data on bar graphs, picture graphs, line plots, and tally charts. Use bar graphs, picture graphs, line plots, and tally charts to answer questions (e.g., how many, most, least) that require interpretation and integration of information presented on the graph. | Draw inferences or make predictions by interpreting information presented on a bar graph, picture graph, line plot, or tally chart (e.g., on the bar graph representing the number of pizzas required for a certain number of people, predict the number of pizzas needed for 20 people). |


| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :--- | :--- | :--- | :--- |
|  | represent the data <br> values, and tally marks <br> are used to represent <br> data on a tally chart. |  |  |  |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

## How is the Initial Precursor related to the Target?

In order to represent and use data, students begin by learning to recognize what is the same and different between familiar items such as color, shape, quantity, size, texture, and pattern. Educators should take care to use attribute words (e.g., circle/square, more/less/same, rough/smooth, red, green, red, green) while defining and demonstrating their meaning. While students do not need to say these words, they do need to learn the meanings. Students will also begin to group two or more items in the same set based on an attribute. As the students group two or more items, the educator will demonstrate the representation in a bar graph or line plot and encourage students to actively participate in its creation.

## Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

How is the Distal Precursor related to the Target? Students actively participate in the creation of bar graphs, picture graphs, line plots, and tally charts by placing representations, x's, or dots for each response to the research question.

## Link to Text-Only Map

M.EE.8.SP. 4 Construct a graph or table from given categorical data, and compare data categorized in the graph or table.


| Map Key |  |
| :--- | :--- |
| IP | Initial Precursor |
| DP | Distal Precursor |
| PP | Proximal Precursor |
| T | Target |
| S | Successor |
| UN | Untested |
| Boxes indicate tested |  |
| nodes |  |

# Mini-Map for M.EE.8.EE. 7 

LEARNING MAPS

## Subject: Mathematics <br> Expressions and Equations (EE) <br> Grade: 8

## Learning Outcome

## DLM Essential Element

M.EE.8.EE. 7 Solve simple algebraic equations with one variable using addition and subtraction.

## Grade-Level Standard

M.8.EE. 7 Solve linear equations in one variable.

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Combine two or more sets of objects or numbers to form a new set. Split one set into multiple sets grouped together by similar characteristics. | Demonstrate understanding of addition by combining the objects of two or more sets and demonstrate understanding of subtraction by removing some objects from a larger set. | Determine the unknown/missing addend (e.g., $8+x=12$ ) or sum (e.g., $4+6=x$ ) when given an equation with addition operation and determine the unknown/missing minuend or subtrahend (e.g., $9-x=16$ ) or the difference (e.g., 13-5 = x) when given an equation with subtraction operation. | Solve linear equations involving addition, subtraction, multiplication, or division operations in one variable (e.g., 8.4 + $x=17.56$ ). | Solve linear inequalities in one variable (e.g., 6 < $8+x$ ). |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

## How is the Initial Precursor related to the Target?

Solving linear equations requires a student to count small units, recognizing that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numbers to label and count the separate sets. The general goal is to explore how the set changes when items are separated out (partitioned) or combined.

## How is the Distal Precursor related to the Target?

As students begin to understand labeling and counting small sets, they begin to use the number sequence and become more adept at tracking individual objects. They can recognize when items are added to a set or when items are taken away. Work on this skill using a variety of sets, labeling and counting the set, and moving items in and out of the set, labeling and counting the set again.

NOTE: Educators can work on the Distal Precursor level using the sets of numbers that students working at the Target level are working with.

## Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

M.EE.8.EE. 7 Solve simple algebraic equations with one variable using addition and subtraction.


| Map Key |  |
| :--- | :--- |
| IP | Initial Precursor |
| DP | Distal Precursor |
| PP | Proximal Precursor |
| T | Target |
| S | Successor |
| UN | Untested |
| Boxes indicate tested |  |
| nodes |  |

# Mini-Map for M.EE.8.EE. 2 

LEARNING MAPS

## Subject: Mathematics <br> Expressions and Equations (EE) <br> Grade: 8

## Learning Outcome

| DLM Essential Element | Grade-Level Standard |
| :--- | :--- |
| M.EE.8.EE.2 Identify a geometric sequence of whole numbers | M.8.EE.2 Use square root and cube root symbols to represent <br> with a whole number common ratio. <br> solutions to equations of the form $x^{2}=p$ and $x^{3}=p$, where $p$ is a <br> positive rational number. Evaluate square roots of small perfect <br> squares and cube roots of small perfect cubes. Know that $V 2$ is <br> irrational. |

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Arrange objects in a specific order by following a specific rule (e.g., arrange objects from the largest to the smallest size). Group like items by attributes such as size, shape, color, and size. Contrast or distinguish objects based on attributes such as shape, size, texture, and numerical pattern. | Recognize patterns (i.e., repeating, growing, shrinking) involving numbers or letters (e.g., $a, b, b, a, b, b . . . ; 2,5,8$, 11...). Identify a sequence as an ordered list of numbers that adheres to a common rule between corresponding numbers (e.g., 2, 4, 6, 8...). | Recognize a growing pattern as a pattern that increases (e.g., 3, 6, 9, 12...) and a shrinking pattern as a pattern that decreases (e.g., 12, 10, 8...). | Recognize a geometric sequence as an ordered list of numbers, such that each term after the first is determined by multiplying or dividing the preceding term by a constant amount (e.g., $2,4,8,16 \ldots$..). | Recognize the recursive rule in geometric sequences by determining how each term in the sequence differs from the preceding term (e.g., the recursive rule in the sequence $2,4,8,16 \ldots$ is "multiply by 2 "). |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

How is the Initial Precursor related to the Target?
In order to recognize geometric patterns, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., "this set has all red objects; this set has all blue," "these fidgets are big; these fidgets are small") and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different.

## Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

## Link to Text-Only Map

M.EE.8.EE. 2 Identify a geometric sequence of whole numbers with a whole number common ratio.


# Mini-Map for M.EE.8.F.1-3 

LEARNING MAPS

## Subject: Mathematics

Functions (F)
Grade: 8

## Learning Outcome

## DLM Essential Element

M.EE.8.F.1-3 Given a function table containing at least 2 complete ordered pairs, identify a missing number that completes another ordered pair (limited to linear functions).

## Grade-Level Standard

M.8.F. 1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
M.8.F. 2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
M.8.F. 3 Interpret the equation $y=m x+b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Form a pair of objects by arranging two objects in a specific order (e.g., form a pair by first placing a pencil and then placing a ruler). Arrange objects by a specified rule (e.g., arrange pencils in order by length). | Recognize a growing pattern as a pattern that increases (e.g., 3, 6, 9, 12...) and a shrinking pattern as a pattern that decreases (e.g., 12, $10,8 \ldots$.. | Communicate understanding that the numbers in the coordinate pair ( $x, y$ ) represent $x$ units left or right on the $x$-axis and $y$ units up or down on the $y$-axis. Communicate the next term in a growing or shrinking pattern, consisting of numerals or letters, by recognizing the core | Generate ordered pairs by recognizing the pattern rules for each coordinate and applying these rules to the $x$ - and $y$-values [e.g., given (1, $3),(2,5),(3,7) \ldots$, the next ordered pair would be $(4,9)]$. | Recognize covariation as the pattern in which two variables or quantities change together. Recognize correspondence as the relationship between each $x$ - and $y$-value. |

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| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :--- | :--- | :--- |
|  |  | unit or the pattern rule <br> and applying it to the <br> pattern (e.g., the <br> pattern rule in the <br> pattern: 3, 6, 9, 12 is <br> "add 3," so the next <br> term in the pattern is <br> $12+3$ equals 15). |  |  |
|  |  |  |  |  |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

## How is the Initial Precursor related to the Target?

In order to understand and work with function tables, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., "this set has all red objects; this set has all blue," "these fidgets are big; these fidgets are small") and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different.

## Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

## Released Testlets

See the Guide to Practice Activities and Released Testlets.

## Using Untested (UN) Nodes

See the document Using Mini-Maps to Plan Instruction

## How is the Distal Precursor related to the Target?

Building on arranging and ordering objects, educators can use some of the other mathematical concepts like working with sets or recognizing a whole and parts to help students identify "same" and "different." For instance, students may create a set and then create a second set that has the same amount. Then, they can change one of the sets to make it different. As students are learning to create and identify sets that are same and different, educators can draw student attention to the various attributes of a set to teach students to order, classify, and contrast the sets. These understandings will then lead to students having the attentional skills to recognize growing and shrinking patterns.

## Link to Text-Only Map

M.EE.8.F.1-3 Given a function table containing at least 2 complete ordered pairs, identify a missing number that completes another ordered pair (limited to linear functions).


## Learning Outcome

## DLM Essential Element

M.EE.8.F. 4 Determine the values or rule of a function using a graph or a table.

## Grade-Level Standard

M.8.F. 4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x, y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

## Linkage Level Descriptions

| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :---: | :---: | :---: | :---: | :---: |
| Form a pair of objects by arranging two objects in a specific order (e.g., form a pair by first placing a pencil and then placing a ruler). | Generate ordered pairs by recognizing the pattern rules for each coordinate and applying these rules to the $x$ - and $y$-values [e.g., given (1, $3),(2,5),(3,7) . .$. , the next ordered pair would be (4, 9)]. Communicate the next term in a growing or shrinking pattern, consisting of numerals or letters, by recognizing the core unit or the pattern rule and applying it to the | Recognize covariation as the pattern in which two variables or quantities change together. Recognize the direction in which two variables change together (e.g., describe that as $x$ increases, $y$ decreases). | Communicate understanding of a function rule from the list of ordered pairs or a graph by determining how $x$ - and $y$-values change and relate to each other (e.g., the slope is $1 / 1$ and each $y$ value is equal to $x$-value +2 , or $y=x+2$ ). | Communicate understanding of a function as a set of ordered pairs or a line on a graph where there exists a relationship between $x$ - and $y$ coordinates, and there are no two ordered pairs with the same input ( $x$-value) and different outputs ( $y$ value). |


| Initial Precursor | Distal Precursor | Proximal Precursor | Target | Successor |
| :--- | :--- | :--- | :--- | :--- |
|  | pattern (e.g., the |  |  |  |
|  | pattern rule in the |  |  |  |
|  | pattern: 3, 6, 9, 12 is |  |  |  |
|  | "add 3," so the next |  |  |  |
|  | term in the pattern is |  |  |  |
|  | $12+3$ equals 15). |  |  |  |

## Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

## How is the Initial Precursor related to the Target?

In order to understand and work with function tables, students begin by learning to notice what is new. The educator draws the students' attention to new objects or stimuli, labels them (e.g., "this set has all red objects; this set has all blue," "these fidgets are big; these fidgets are small") and the student observes, feels, or otherwise interacts with them. Educators encourage students to begin placing like objects together, drawing attention to the characteristics that make an item the same or different. Educators provide sorting activities that allow learners to isolate specific attributes while recognizing likenesses and differences among objects. Educators also provide activities that reinforce the skill of ordering (e.g., arrangement of objects from largest to smallest, sequencing daily events, and counting).

## How is the Distal Precursor related to the Target?

As student attention to objects and details develops, educators can extend their attention by providing experience with finding and creating simple patterns using objects and moving to symbols (e.g., numerals). Educators should take care to start with simple patterns (e.g., 1-2-1-2) and take advantage of the symbols that are already being used in the classroom. Educators should demonstrate how students can create and identify the pattern/rule (e.g., using colored cubes, the student creates a line of 5 cubes, the educator then creates a matching set and explains what to do to follow the student's pattern. Then, the student generates a third matching set. If the order is not followed, it is a good teaching opportunity to talk about why it doesn't fit the pattern). Learning to identify the rule of patterns will help students extend their thinking across patterns. As students work on identifying pattern rules, educators can also begin to demonstrate how rules can be used with ordered pairs (e.g., see example below). Provide students lots of opportunities to apply rules to create their own examples of ordered pairs.

| Input | Rule | Output |
| :---: | :---: | :---: |
| 5 | +1 | 6 |
| 4 | +1 | 5 |
| 7 | +1 | 8 |
| 1 | +1 |  |


| Input | Rule | Output |
| :---: | :---: | :---: |
| 5 | -2 | 3 |
| 4 | -2 | 2 |
| 7 | -2 |  |
| 9 | -2 | 7 |

## Instructional Resources

| Released Testlets |
| :--- |
| See the Guide to Practice Activities and Released Testlets. |
| Using Untested (UN) Nodes |
| See the document Using Mini-Maps to Plan Instruction. |

M.EE.8.F. 4 Determine the values or rule of a function using a graph or a table.


