

# Mathematics Glossary

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<b>absolute value</b>	The distance of a number from the <i>origin</i> .
<b>absolute value function</b>	A <i>function</i> where the outputs of nonnegative (i.e., $x \geq 0$ ) inputs equal the inputs, and the outputs of negative (i.e., $x < 0$ ) inputs are the <i>opposites</i> of the inputs.
<b>acute angle</b>	An angle whose <i>measure</i> is less than 90 degrees.
<b>acute triangle</b>	A triangle in which each angle <i>measures</i> less than 90 degrees.
<b>addend</b>	A number that is added to another (e.g., In the equation, $4 + 6 = 10$ , 4 and 6 are addends).
<b>addition strategies</b>	<i>Strategies</i> that help students to add two or more than two numbers (e.g., counting all, forward counting, making groups of tens and ones, <i>doubling</i> a number, etc.).
<b>addition word problem</b>	A hypothetical situation expressed in words that requires students to find an unknown quantity using an addition operation or <i>strategy</i> .
<b>additive comparison</b>	A numerical comparison that indicates how much more or less one quantity is than another (e.g., numbers, <i>length measures</i> ).
<b>additive inverse(s)</b>	Two numbers whose <i>sum</i> is zero are additive inverses of each other (i.e., <i>opposites</i> ).
<b>adjacent angles</b>	Two <i>angles</i> in the same plane that share a common <i>vertex</i> and a common <i>ray</i> .
<b>adjacent objects</b>	Two objects that are placed close to each other (e.g., touching).
<b>algebra tiles</b>	Mathematical <i>manipulatives</i> in the form of small <i>squares</i> , large squares, and <i>rectangles</i> that help students to develop algebraic thinking in constructive ways.
<b>analyze</b>	To study or examine the structure of a mathematical situation (e.g., relationship between an answer and the problem's context).
<b>angle</b>	A figure formed by two <i>rays</i> sharing one <i>endpoint</i> .
<b>apply</b>	To use prior <i>understanding</i> , conceptual knowledge, or procedural skill to solve problems or to make sense of novel situations.
<b>area</b>	A two-dimensional quantity representing the amount of space in a surface.
<b>area model (fractions)</b>	A pictorial representation ( <i>model</i> ) of <i>fractions</i> as parts of <i>areas</i> , such as fraction <i>circles</i> or rectangular fraction bars.

<b>arithmetic sequence</b>	A <i>sequence</i> of numbers such that each <i>term</i> after the first term can be obtained from the previous term by adding or subtracting a constant term.
<b>arithmetic series</b>	The <i>sum</i> of an <i>arithmetic sequence</i> .
<b>array</b>	A rectangular arrangement of objects or numbers in rows and columns.
<b>associative property of addition</b>	The <i>sum</i> of three or more numbers is the same regardless of the grouping of the addends (e.g., $10 + (4 + 2) = (10 + 4) + 2$ ).
<b>attribute</b>	A characteristic or property of an object such as <i>length</i> , weight, capacity, time, number of sides, shape, or color.
<b>auditory pattern</b>	A set of sounds that are arranged following a rule (e.g., clap, snap, snap, clap, snap, snap, clap, snap, snap).
<b>automaticity</b>	The ability to recall or generate an answer quickly and effortlessly from memory or by applying factual knowledge.
<b>base ten blocks</b>	Mathematical <i>manipulatives</i> representing ones (cubes), tens (sticks), and hundreds (flats), useful for developing <i>understanding</i> of <i>place value</i> and <i>modeling</i> mathematical concepts such as addition and subtraction.
<b>base number</b>	A real number $b$ in the <i>expression</i> $b^n$ that is multiplied $n$ times (e.g., In the power $3^4$ , 3 is the base number that is multiplied four times (i.e., $3 \times 3 \times 3 \times 3$ )).
<b>bar graph</b>	A graphical display of categorical data using rectangular bars of varied heights.
<b>bivariate data</b>	Data involving two <i>variables</i> , usually related to each other (e.g., height and weight of football players).
<b>box plot</b>	A visual display that summarizes data using a 'box and whiskers' format to show the minimum and maximum values (ends of the whiskers), <i>quartiles</i> (ends of the box), <i>interquartile range</i> ( <i>length</i> of the box), and <i>median</i> ( <i>line</i> through the box). It provides useful information regarding the spread of the data and any <i>outliers</i> that the data may have.
<b>cardinality</b>	<i>Understanding</i> that the last number counted equals the number of objects that have been counted thus far.
<b>Cartesian product</b>	The <i>set</i> of all ordered pairs generated when each element from one set is matched with each element from a second set (i.e., $A \times B = \{(a, b) \mid a \in A \text{ } \forall b \in B\}$ ).
<b>Cavalieri's principle</b>	If two solids of equal altitude are divided with the same plane and the <i>length</i> of the cross-sections are also same, then the two objects have the same <i>volume</i> .

<b>central angle</b>	An <i>angle</i> where the <i>vertex</i> of which is the center of a <i>circle</i> and the <i>rays</i> of which pass through <i>points</i> on the <i>circumference</i> of the <i>circle</i> .
<b>change problems</b>	Mathematical situations involving a change in the initial amount of a quantity. Such problems involve three quantities: an initial amount, a change amount, and the resulting amount.
<b>chord</b>	A <i>line segment</i> whose <i>endpoints</i> lie on the <i>circumference</i> of a <i>circle</i> .
<b>circle</b>	A <i>set</i> of <i>points</i> in a plane equidistant from a given point (i.e., center). A closed plane curve generated by the trace of a point moving in such a way that its distance from a fixed point (i.e., center) remains constant (i.e., <i>radius</i> ).
<b>circular arc</b>	A segment of the <i>circumference</i> of a <i>circle</i> .
<b>circumference</b>	The <i>perimeter</i> of a circular <i>area</i> .
<b>closed set</b>	A <i>set</i> is closed under a given operation if and only if the operation on two elements of the set produces an element of the same set (e.g., The set $X = \{2, 4, 6, 8, 10\}$ is closed under subtraction).
<b>coefficient</b>	A number or symbol (e.g., 2, $e$ ) multiplied by a <i>variable</i> (e.g., $x$ ) in an algebraic <i>term</i> (e.g., $2x$ ).
<b>combine problems</b>	Mathematical situations involving the combination of two discrete <i>sets</i> or splitting of one set into two discrete sets. See <i>part-part-whole problems</i> .
<b>combination (probability)</b>	A subset of items selected from a given <i>set</i> without regard to order (e.g., 1, 2, 3 and 3, 2, 1 are the same combination).
<b>common factor</b>	A number that is a <i>factor</i> for two or more numbers (e.g., 2 is a common factor of 4, 8, and 12).
<b>commutative property of addition</b>	The <i>sum</i> of two or more numbers is the same regardless of the order of the addends (e.g., $4 + 2 = 2 + 4$ ).
<b>commutative property of multiplication</b>	The <i>product</i> of two or more numbers is the same regardless of the order of the <i>factors</i> (e.g., $4 \times 3 = 3 \times 4$ ).
<b>compare problems</b>	Mathematical situations involving comparisons of two discrete <i>sets</i> of objects (e.g., to identify smaller or larger quantities, <i>differences</i> between quantities, or <i>ratio</i> relationships).
<b>complement of a set</b>	A <i>set</i> of elements not in the given set but in a universal set.

<b>complementary angles</b>	Two <i>angles</i> whose <i>sum</i> is 90 degrees.
<b>complex conjugate</b>	A pair of <i>complex numbers</i> where the real parts are the same, and the <i>imaginary</i> parts are of equal magnitude but have opposite signs.
<b>complex number</b>	Numbers consisting of both real and <i>imaginary</i> parts that can be expressed as $a + bi$ .
<b>compose (numbers, shapes)</b>	Combine two shapes to form a new shape (e.g., combine two <i>squares</i> to form a <i>rectangle</i> ), or combine smaller numbers to form a larger number (e.g., combine 4 and 5 to make 9).
<b>composite number</b>	A number that has <i>factors</i> other than one and itself.
<b>compound event</b>	An event comprising of two or more <i>simple events</i> (e.g., getting heads in a coin toss and an <i>even number</i> when rolling a die).
<b>conceptual subitizing</b>	To count the number of objects in a <i>set</i> by consciously <i>partitioning</i> that set into smaller subsets that can be <i>perceptually subitized</i> .
<b>concrete pattern</b>	A set of concrete manipulatives, such as pattern blocks, coins, or colored counters, that are arranged following a rule  (e.g.,  ).
<b>conditional probability</b>	The <i>probability</i> that an event will occur given that another event occurs.
<b>conditional relative frequency</b>	The frequency of one event occurring given that another event occurs.
<b>congruent angle</b>	<i>Angles</i> with the same <i>angle measure</i> .
<b>congruent figure</b>	A figure that is an exact copy of another figure, (i.e., where corresponding sides and <i>angles</i> are <i>congruent</i> ) OR A two-dimensional figure is congruent to another figure if the second can be obtained from the first by a sequence of <i>rotations</i> , <i>reflections</i> , and <i>translations</i> .
<b>conservation of number</b>	The number of objects in a <i>set</i> remains the same no matter how they are arranged or counted.
<b>conservation of shape</b>	The shape of an object remains the same no matter how an object is <i>transformed</i> (i.e., <i>translated</i> , <i>reflected</i> , or <i>rotated</i> ).

<b>construct</b>	Generate mathematical <i>representations</i> , including but not limited to drawings (e.g., <i>lines</i> , <i>angles</i> , shapes, and diagrams), symbolic <i>equations</i> or <i>expressions</i> , and graphical displays (e.g., graphs, tables, and charts).
<b>coordinate plane</b>	A plane on which <i>points</i> are located and plotted using <i>x</i> - and <i>y</i> -coordinates.
<b>core (repeating) unit</b>	The shortest string of symbols forming a unit that repeats in a repeating pattern (e.g., in the repeating pattern 1, 1, 2, 1, 1, 2, 1, 1, 2, the core unit is 1, 1, 2).
<b>correlation coefficient</b>	A number between $-1$ and $+1$ that <i>measures</i> the degree to which two <i>variables</i> are linearly related.
<b>correspondence (function)</b>	A view of <i>function</i> that focuses on the mapping of each value of one variable with exactly one value for another variable.
<b>cosecant</b>	The <i>ratio</i> of the <i>length</i> of the <i>hypotenuse</i> to the length of the side lying opposite of an <i>acute angle</i> in a right triangle.
<b>cosine</b>	The <i>ratio</i> of the <i>length</i> of the side lying adjacent to an <i>acute angle</i> to the length of the <i>hypotenuse</i> in a right triangle.
<b>cotangent</b>	The <i>ratio</i> of the <i>length</i> of the side lying adjacent to an <i>acute angle</i> to the side opposite of that <i>acute angle</i> in a right triangle.
<b>covariation (function)</b>	A view of <i>function</i> that focuses on the way two quantities or variables change together.
<b>cube root</b>	The number whose cube is a given number.
<b>cube root function</b>	A <i>function</i> where the outputs are the <i>cube roots</i> of the inputs.
<b>decade numerals</b>	A numeral among 10, 20, 30, 40, 50, 60, 70, 80, or 90 that can be expressed as one, two, three, four, five, six, seven, eight, or nine tens and zero ones, respectively.
<b>decompose number</b>	Express a number as the <i>sum</i> of smaller numbers.
<b>define</b>	To <i>explain</i> or <i>describe</i> clearly and completely using mathematically appropriate language.
<b>degree (polynomial)</b>	The greatest <i>exponent</i> of a <i>variable</i> (or <i>sum</i> of exponents of multiplied variables) in a <i>polynomial</i> .
<b>demonstrate</b>	To show <i>understanding</i> of a concept by physically <i>constructing</i> a concrete <i>representation</i> . For example, using counters to show two plus three equals five or showing that the <i>volume</i> of a cylinder is three times the volume of a cone by filling them with rice.

<b>denominator</b>	The number below or after the fraction bar in a <i>fraction</i> representing the number of equal parts into which a whole is divided. The name for what is being counted by a fraction (e.g., denominator of 4 means one is counting fourths).
<b>dependent variable</b>	A <i>variable</i> in a <i>function</i> whose value is influenced by the value of another variable (i.e., <i>independent variable</i> ).
<b>describe</b>	To communicate or portray the meaning of something using verbal or written explanations, concrete <i>models</i> , gestures, assistive devices, etc.
<b>diagonal</b>	A <i>line segment</i> joining two nonconsecutive vertices of a <i>polygon</i> .
<b>diameter</b>	A <i>line segment</i> that passes through the center of and intersects at two <i>points</i> the <i>circumference</i> of a <i>circle</i> .
<b>difference</b>	The result obtained by subtracting one number from another.
<b>dilation</b>	A <i>transformation</i> that enlarges or reduces a <i>line</i> or shape by a given scale factor.
<b>direct comparison</b>	The process of comparing objects without using a measuring tool (e.g., by lining up, matching, visually <i>estimating</i> ).
<b>distributive property</b>	Multiplying a <i>sum</i> or <i>difference</i> by a given number yields the same result as multiplying each addend by the number and then adding the <i>products</i> (e.g., $a(b + c) = a \times b + a \times c$ ).
<b>dividend</b>	A number that is being divided by another number (i.e., <i>divisor</i> ).
<b>divisor</b>	A number that divides another number (i.e., <i>dividend</i> ).
<b>domain of function</b>	The <i>set</i> of all possible inputs (i.e., <i>independent variable</i> , <i>x-values</i> ).
<b>dot plot</b>	A graphical display of data above a <i>number line diagram</i> , where each data point (i.e., dot) is plotted above the corresponding value on the number line.
<b>doubling strategy</b>	To count, add, or subtract by doubling a given number (e.g., adding 8 and 9 by doubling 8 and adding 1 to it). See <i>strategy</i> .
<b>edge</b>	A line segment joining two vertices or corners of a two-dimensional shape or two faces of a three-dimensional object. See <i>vertex</i> .
<b>ellipse</b>	A closed plane curve generated by the trace of a <i>point</i> moving in such a way that the <i>sum</i> of its distances from two fixed points (i.e., <i>foci</i> ) remains constant.
<b>equal group problems</b>	Problems that require students to count or organize quantities using equal groups to find the number of groups, group size, or <i>product</i> .

<b>equal sign</b>	A sign used in an <i>equation</i> to represent an equivalent relationship between <i>expressions</i> .  <u>Note:</u> It should be recognized as more than a signal to perform a given computation or a signal that the answer to a problem comes next.
<b>equation</b>	A mathematical sentence involving two equivalent <i>expressions</i> and an <i>equal sign</i> .
<b>equilateral triangle</b>	A triangle with three congruent sides.
<b>equivalent expressions</b>	Numerical or algebraic <i>expressions</i> that may include different mathematical phrases but represent the same value (e.g., $3a + 3b = 3(a + b)$ , $3 + 4 + 7 = 7 + 7$ ).
<b>equivalent fractions</b>	<i>Fractions</i> with different <i>numerators</i> and <i>denominators</i> that represent the same <i>ratio</i> or relative amount (e.g., $\frac{2}{4}$ and $\frac{4}{8}$ both are equal to $\frac{1}{2}$ ).
<b>estimate</b>	Use mental and visual information to <i>measure</i> an <i>attribute</i> (e.g., <i>length</i> , <i>area</i> , <i>volume</i> , etc.) or make comparisons without using standard measurement tools.
<b>even function</b>	A <i>function</i> whose graph is symmetric about the <i>y-axis</i> .
<b>even number</b>	An <i>integer</i> that is a <i>multiple</i> of two, including zero.
<b>expanded form</b>	A representation of a number as the <i>sum</i> of the values of the digits in the number.
<b>experimental probability</b>	The <i>ratio</i> of the number of times an event occurs to the total number of times the activity (i.e., experiment) is conducted.
<b>explain</b>	To make clear one's thinking, <i>understanding</i> , and reasoning to others by providing justifications.
<b>exponent</b>	A real number that indicates how many times a <i>base number</i> is to be multiplied by itself (e.g., $3^4 = 3 \times 3 \times 3 \times 3$ , where 3 is the base number and 4 is the exponent).
<b>exponential function</b>	A <i>function</i> ( $f(x) = a \times bx$ , where $b > 0$ ) in which a constant change in the <i>independent variable</i> (input) gives a same <i>proportional</i> change in the <i>dependent variable</i> (output) (e.g., The depreciation in the value of a car as a function of time is an exponential function represented as $y = 20,000(\frac{1}{2})t$ , where $t$ = time in years)
<b>expression</b>	A mathematical phrase including numbers and/or <i>variables</i> with or without operations but not an <i>equal sign</i> (e.g., 5, $8 + 7$ , $7x - y$ , $4z$ ).
<b>factor</b>	Numbers that are multiplied together to yield a given number.

<b>fraction</b>	A number expressed as the <i>quotient</i> of two <i>integers</i> , in the form $\frac{a}{b}$ ( $b \neq 0$ ), where $a$ is the number of parts (i.e., the <i>numerator</i> ) when the whole is divided into $b$ equal parts (i.e., the <i>denominator</i> ).
<b>function</b>	A relation in which each input ( $x$ -values or <i>domain</i> ) is paired with one and only one output ( $y$ -values or <i>range</i> ). If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ .
<b>function rule</b>	A rule that describes the relationship between the inputs and outputs of a specified <i>function</i> .
<b>geometric sequence</b>	A <i>sequence</i> of numbers such that each <i>term</i> after the first term can be obtained from the previous term by multiplying by a constant term.
<b>geometric series</b>	The <i>sum</i> of a <i>geometric sequence</i> .
<b>greatest common factor</b>	A number that is the largest <i>common factor</i> for two or more numbers.
<b>growing pattern</b>	A <i>pattern</i> that grows in value or size from term to term (e.g., 1, 2, 4, 8).
<b>hierarchical inclusion</b>	The <i>understanding</i> that numbers are nested inside of each other (e.g., numbers grow by one with each count).
<b>histogram</b>	A graphical display of continuous data using rectangular bars of different heights.
<b>horizontal (line)</b>	A straight <i>line parallel</i> to the horizon (or parallel to the <i>x-axis</i> on a graph).
<b>horizontal asymptote</b>	A <i>horizontal line</i> that the graph of a <i>function</i> approaches as $x$ tends to positive infinity or negative infinity.
<b>horizontal compression</b>	The squeezing of a graph or figure towards the <i>y-axis</i> .
<b>horizontal stretching</b>	The stretching of a graph or figure away from the <i>y-axis</i> .
<b>hyperbola</b>	A closed plane curve generated by the trace of a <i>point</i> moving in such a way that the distances between any point to a fixed point (i.e., focus) and a fixed straight <i>line</i> (i.e., directrix) are always in the same <i>ratio</i> .
<b>hypotenuse</b>	The side opposite to the <i>right angle</i> in a right triangle.
<b>identity property of addition</b>	The <i>sum</i> of any number and zero equals the original number.

<b>identity property of multiplication</b>	The <i>product</i> of any number and one equals the original number.
<b>imaginary number</b>	Numbers written in the form $b(i)$ , where $b$ is a real number and $i$ is an imaginary unit.
<b>improper fraction</b>	A <i>fraction</i> where the <i>numerator</i> is larger than the <i>denominator</i> .
<b>independent variable</b>	A <i>variable</i> in a <i>function</i> whose value influences the value of another variable (i.e., <i>dependent variable</i> ).
<b>informal/non-standard units of measurement</b>	Units of <i>measurement</i> that are not <i>standard units of measurement</i> and not part of the U.S. customary or metric measurement systems.
<b>integers</b>	The <i>set</i> of <i>whole numbers</i> and their <i>opposites</i> (i.e., $\{-2, -1, 0, 1, 2\}$ ).
<b>interpret</b>	To form <i>understanding</i> of a concept or the solution to a problem, such as by relating the solution to the problem's context.
<b>interquartile range</b>	The <i>difference</i> between the <i>upper quartile</i> and <i>lower quartile</i> .
<b>intersecting lines (line segments)</b>	<i>Lines</i> or <i>line segments</i> that have at least one <i>point</i> in common.
<b>intersection of sets</b>	The <i>set</i> that contains all the elements of one set that also belong to another set.
<b>irrational number</b>	Any real number that cannot be expressed as a <i>fraction</i> (e.g., <i>pi</i> ( $\pi$ ); roots of <i>prime numbers</i> ; and non-repeating, non-terminating decimals).
<b>isosceles triangle</b>	A triangle with at least two equal sides.
<b>iterate (unit)</b>	Repeat a <i>unit</i> (e.g., a fractional part or a unit of <i>measure</i> ) to make the whole or to measure a given object (e.g., in the <i>fraction</i> $\frac{3}{2}$ , $\frac{1}{2}$ of the unit is repeated three times).
<b>join problems</b>	<i>Change problems</i> where the change amount is "added" to the initial amount. In these problems, the initial amount, change amount, or resulting amount is unknown.
<b>justify</b>	To show or prove one's thinking or reasoning (e.g., a solution, <i>strategy</i> , <i>representation</i> , or rationale pertaining to a mathematical situation).
<b>know</b>	To remember or have a clear <i>understanding</i> of something.
<b>law of cosines</b>	If $a$ , $b$ , and $c$ are the sides of a triangle and $C$ is the <i>angle</i> opposite side $c$ , then $c^2 = a^2 + b^2 - 2ab \times \cos(\angle C)$ . See <i>cosine</i> .

<b>law of sines</b>	If $a$ , $b$ , and $c$ are the sides of a triangle and $A$ , $B$ , and $C$ are the <i>angles</i> opposite sides $a$ , $b$ , and $c$ , then $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ . See <i>sine</i> .
<b>least common denominator</b>	The smallest positive <i>integer</i> that is a common <i>multiple</i> of all <i>denominators</i> of two or more <i>fractions</i> .
<b>least common multiple</b>	The smallest number that is a <i>multiple</i> of two or more numbers.
<b>length</b>	The distance between the two <i>points</i> that define a <i>line segment</i> .
<b>length model</b>	A linear representation ( <i>model</i> ) of <i>fractions</i> as <i>line segments</i> , where the value of each fraction equals its distance from zero on a <i>number line diagram</i> or <i>length</i> of the segment/rod when using Cuisenaire rods (a type of <i>manipulative</i> ).
<b>likelihood (probability)</b>	The chance that an event will occur.
<b>line</b>	A straight line that extends infinitely in two directions.
<b>line plot</b>	A graphical display of data above a <i>number line diagram</i> , where each data point (i.e., $\times$ ) is plotted above the corresponding value on the number line.
<b>line segment</b>	A part of a <i>line</i> that is bounded by two <i>endpoints</i> .
<b>line symmetry</b>	A geometric figure is said to have line symmetry if, when the shape is (virtually) folded on a <i>line</i> , the two halves are exactly the same.
<b>linear function</b>	A <i>function</i> represented by an <i>equation</i> $f(x) = mx + b$ and whose graph is a straight <i>line</i> .
<b>logarithmic function</b>	The inverse of the <i>exponential function</i> , (i.e., a <i>function</i> in which a <i>proportional</i> change in the <i>independent variable</i> (input) gives a constant change in the <i>dependent variable</i> (output)).
<b>lower quartile</b>	The <i>median</i> of the lower half of a data set.
<b>magnitude</b>	The absolute value of a number or a number's distance from zero.
<b>manipulatives</b>	Concrete objects, such as blocks, counters, beads, puzzles, etc., that can be used within instruction to <i>represent</i> mathematical concepts in a concrete manner.
<b>margin of error</b>	The amount of error caused by <i>random sampling</i> .

<b>marginal relative frequencies</b>	Total frequencies for rows or columns in a two-way table.
<b>mass</b>	The amount of matter in an object.
<b>mathematize</b>	To <i>interpret</i> a real-world situation by organizing, <i>sorting</i> , and coordinating information to develop a mathematical <i>representation</i> of a given problem.
<b>matrix</b>	A rectangular <i>array</i> of numbers, symbols, or <i>expressions</i> .
<b>mean</b>	A point on which a distribution would balance. It is calculated by adding all numbers in the <i>set</i> of scores and dividing the <i>sum</i> by the total number of scores.
<b>mode</b>	The data value that appears most frequently in a given <i>set</i> of data.
<b>measurable attributes</b>	<i>Attributes</i> of objects that can be <i>measured</i> (e.g., <i>length</i> , <i>mass</i> , <i>volume</i> , capacity, etc.).
<b>measure</b>	A number that indicates an <i>attribute</i> value of an object in reference to some <i>standard</i> or <i>informal/non-standard units of measurement</i> .
<b>median (data)</b>	The number at the center of a data distribution that separates the data <i>set</i> into higher and lower halves.
<b>median of a triangle</b>	A <i>line segment</i> joining a <i>vertex</i> of a triangle and the <i>midpoint</i> of the side opposite of that vertex.
<b>mixed number</b>	A number consisting of a <i>whole number</i> and a proper <i>fraction</i> , (e.g., $4\frac{1}{3}$ ).
<b>models</b>	(n.) <i>Representations</i> that show how students are thinking about, describing, or <i>interpreting</i> a mathematical situation (e.g., <i>equations</i> , graphs, tables, diagrams, etc.).
<b>model</b>	(v.) To <i>demonstrate</i> a mathematical concept in a specific way (e.g., to model relationship between addition and subtraction).
<b>multiple</b>	The <i>product</i> of a given number and a <i>whole number</i> .
<b>multiplicative comparison</b>	A numerical comparison that indicates how many times one quantity is larger than another or what <i>fractional</i> part one quantity is of another.
<b>negative exponent property</b>	Negative <i>exponents</i> indicate reciprocation. They can be calculated by determining the value as if the exponent was positive and then taking the reciprocal of that result: $aa^{-bb} = \frac{1}{ab}$ , $aa \neq 0$ , and $\frac{1}{aa^{-bb}} = aabb$ .

<b>non-defining attributes</b>	<i>Attributes</i> such as color, orientation, or size that do not define the geometric aspects of a shape.
<b>non-linear function</b>	A <i>function</i> that is not linear, not represented by $f(x) = mx + b$ , and whose graph is not a straight <i>line</i> .
<b>normal distribution</b>	A distribution of data for which the <i>mean</i> , <i>median</i> , and <i>mode</i> are equal and divide the data in half. The resulting <i>histogram</i> looks like a bell-shaped curve.
<b>number line diagram</b>	A straight <i>line model</i> , <i>partitioned</i> into equal segments, on which real numbers can be represented.
<b>number pattern</b>	A <i>pattern</i> that can be described in terms of numerical relationships.
<b>number sequence pattern</b>	The <i>pattern</i> in the number system that determines the number names for numbers greater than 20 (e.g., numbers from 21 to 29 follow the pattern of naming the decade number, twenty, and then the digit name, <i>one</i> , <i>two</i> , <i>three</i> , <i>four</i> , <i>five</i> , <i>six</i> , <i>seven</i> , <i>eight</i> , or <i>nine</i> ).
<b>numerator</b>	The number above or before the fraction bar in a <i>fraction</i> representing the number of equal parts. The name for how many parts are being counted by a fraction (e.g., numerator of 3 means one is counting three parts).
<b>obtuse angle</b>	An <i>angle</i> whose <i>measure</i> is greater than 90 degrees.
<b>obtuse triangle</b>	A triangle in which one <i>angle measures</i> more than 90 degrees.
<b>odd functions</b>	A <i>function</i> whose graph is symmetric with respect to the <i>origin</i> .
<b>odd number</b>	An <i>integer</i> that is not a <i>multiple</i> of two or cannot be divided by two.
<b>one-step word problems</b>	Word problems that can be solved using a single mathematical operation (e.g., <i>change problems</i> , <i>combine problems</i> , <i>compare problems</i> ).
<b>opposite number</b>	Two numbers with the same magnitude but have different signs (i.e., positive, negative) (e.g., 3 and -3).
<b>order irrelevance</b>	When counting a <i>set</i> of objects, it does not matter where one starts or in what order one counts, as long as one counts every object once and only once.
<b>ordered (coordinate) pairs</b>	A set of number pairs ( $x, y$ ) that indicates the position of a point on a graph, where $x$ represents the number of units left or right of the origin and $y$ represents the number of units up or down from the origin

<b>ordered pair</b>	Two numbers, written in the form $(a, b)$ , that define the location of a <i>point</i> on a coordinate plane. The first number (i.e., $a$ ) tells how far from the origin the point is on the horizontal axis ( $x$ -axis) and the second number (i.e., $b$ ) tells how far from the origin the point is on the vertical axis ( $y$ -axis).
<b>origin</b>	The <i>point</i> where the axes of the <i>coordinate plane</i> intersect, at which point both coordinates equal zero (i.e., the point $(0, 0)$ ).
<b>outlier</b>	A data point that lies outside the <i>range</i> of most of the other values in a <i>set</i> of data.
<b>parabola</b>	A plane curve generated by the trace of a <i>point</i> moving in such a way that its distances from a fixed point (i.e., focus) and a fixed <i>line</i> (i.e., directrix) are equal.
<b>parallel lines (line segments)</b>	<i>Lines</i> or <i>line segments</i> that are equal distance apart and never intersect.
<b>pattern rule</b>	A rule that indicates how a pattern is formed.
<b>part-part-whole problems</b>	Problems involving two parts that are combined into one whole. In these problems, either the whole, one of the parts, or both of the parts is unknown.
<b>partition</b>	The division of an object into two or more parts or of a <i>set</i> into two or more distinct subsets.
<b>Pascal's triangle</b>	A triangular <i>array</i> generated by adding two elements in one row to calculate each element of the next row. The completed triangle depicts several arithmetic <i>patterns</i> , and the rows list the binomial <i>coefficients</i> for <i>polynomial products</i> .
<b>pattern</b>	A <i>set</i> of numbers, objects, or shapes that are arranged following a rule.
<b>pattern rule</b>	A rule that indicates how a pattern is formed.
<b>percent</b>	A rate per hundred, derived from a <i>ratio</i> with a <i>denominator</i> of 100 and expressed as 100 times the value of the ratio.
<b>perceptual subitizing</b>	Recognizing the number of objects without consciously using any mental or mathematical <i>strategies</i> . See <i>subitize</i> .
<b>perfect cubes</b>	A number that can be expressed as the cube of a <i>whole number</i> .
<b>perfect squares</b>	A number that can be expressed as the square of a <i>whole number</i> .
<b>perimeter</b>	The distance that surrounds a plane <i>area</i> .
<b>permutation</b>	A subset of items selected from a given <i>set</i> that acknowledges different orderings of the same elements (e.g., 1, 2, 3 and 3, 2, 1 are different permutations).

<b>perpendicular bisector</b>	A <i>line</i> which divides a <i>line segment</i> into two equal parts and intersects the line segment at an <i>angle</i> measuring 90 degrees (i.e., a <i>right angle</i> ).
<b>perpendicular lines (line segments)</b>	Two <i>lines</i> or <i>line segments</i> that meet at a 90 degree <i>angle</i> (i.e., a <i>right angle</i> ).
<b>pi (<math>\pi</math>)</b>	An <i>irrational number</i> that represents the <i>ratio</i> of the <i>circumference</i> of a <i>circle</i> to its <i>diameter</i> .
<b>pictorial pattern</b>	A set of pictures, drawings, or geometric shapes that are arranged following a rule (e.g., ▲■▲■▲■).
<b>picture graph</b>	A graph that uses pictures or drawings to <i>represent</i> data.
<b>pie chart</b>	A circular chart divided into <i>sectors</i> , where each sector is <i>proportional</i> in size to the data it represents.
<b>place value</b>	The numerical value of a digit in a numeral.
<b>plane</b>	A flat, two-dimensional surface that extends infinitely in all directions. Any three non-collinear points determine a plane.
<b>point</b>	A precise location or place on a plane or in space, usually represented by a dot.
<b>polygon</b>	A closed, two-dimensional figure with all straight sides.
<b>polynomial</b>	A mathematical <i>expression</i> constructed from <i>variables</i> , constants, or both using addition, subtraction, multiplication, and non-negative <i>integer exponents</i> .
<b>population (statistics)</b>	A large group of people for which <i>statistical inferences</i> can be made.
<b>predict</b>	The process of using specific knowledge or <i>understanding</i> to <i>estimate</i> what will come next (e.g., predicting the next number in a <i>sequence</i> ).
<b>prime numbers</b>	Any <i>whole number</i> (other than 0 or 1) that has only one <i>set of factors</i> (i.e., 1 and the number itself). Examples include 2, 5, and 7.
<b>probability</b>	A <i>measure</i> of how likely an event is to occur.
<b>product</b>	The quantity obtained by multiplying two or more numbers or algebraic <i>expressions</i> together.
<b>product of powers property</b>	When two numbers with the same base and different <i>exponents</i> are multiplied, the <i>product</i> equals the same <i>base number</i> raised to the <i>sum</i> of the exponents (e.g., $5^2 \times 5^4 = 5^{2+4} = 5^6$ ).

<b>proportion</b>	A statement of equality between two <i>ratios</i> .
<b>Pythagorean theorem</b>	The relationship between the sides of a right triangle: For a right triangle with legs $a$ and $b$ and <i>hypotenuse</i> $c$ , $a^2 + b^2 = c^2$ .
<b>quadrant</b>	One of the four infinite regions formed when a <i>coordinate plane</i> is divided by the $x$ - and $y$ -axis.
<b>quadrilateral</b>	A <i>polygon</i> with four sides.
<b>quartiles</b>	The values that divide a <i>set</i> of data points into four parts such that each part contains the same number of data points.
<b>quotient</b>	The quantity obtained when one divides one number or <i>expression</i> by another.
<b>quotient of powers property</b>	When two numbers with the same base and different <i>exponents</i> are divided, the <i>quotient</i> equals the same <i>base number</i> raised to the <i>difference</i> of exponents (e.g., $\frac{6^5}{6^2} = 6^{5-2} = 6^3$ ).
<b>radian</b>	A unit for measuring <i>angles</i> defined as the <i>ratio</i> of the <i>circumference</i> of a <i>circle</i> to its <i>radius</i> .
<b>radius</b>	A straight <i>line segment</i> from the <i>circumference</i> of a <i>circle</i> to its center.
<b>random sample</b>	A <i>sample</i> where each member in a <i>population</i> has an equal chance of being selected for the sample.
<b>range of data</b>	The <i>difference</i> between the highest and the lowest data points in a given data <i>set</i> .
<b>range of function</b>	The <i>set</i> of all possible outputs ( $y$ -values) of a <i>function</i> , given a specified <i>domain</i> for the function.
<b>ratio</b>	A relationship between two quantities, often expressed as $a$ to $b$ , $a:b$ , or $\frac{a}{b}$ .
<b>rational numbers</b>	The <i>set</i> of numbers including <i>whole numbers</i> , <i>integers</i> , and <i>fractions</i> .
<b>ray</b>	A part of a <i>line</i> that begins at one <i>point</i> and extends infinitely in one direction.
<b>real numbers</b>	The set of numbers that are not imaginary numbers, comprising rational numbers (e.g., 0, 1, 2, $\frac{3}{4}$ , 0.125, etc.) and irrational numbers (e.g., $\pi$ , $\sqrt{3}$ , $\sqrt{10}$ , etc.).
<b>recognize</b>	To identify something either by naming, selecting, or pointing as an indication of one's <i>understanding</i> of a particular concept.

<b>rectangle</b>	A <i>quadrilateral</i> with congruent opposite sides (i.e., parallelogram) and four <i>right angles</i> .
<b>recursive rule (sequences)</b>	A rule that indicates how consecutive terms in a <i>sequence</i> are determined (e.g., arithmetic sequence, geometric sequence).
<b>rectangular prism</b>	A three-dimensional object with six rectangular faces.
<b>reflection</b>	A <i>transformation</i> that flips a <i>point</i> , <i>line</i> , or shape across a line of reflection.
<b>regular polygon</b>	A <i>polygon</i> with all congruent sides and <i>congruent angles</i> .
<b>relative frequency</b>	The <i>ratio</i> of the number of times an outcome occurs to the total number of trials.
<b>remainder</b>	The amount left over after dividing two <i>integers</i> .
<b>repeated addition strategy</b>	To add the same addend a given number of times to solve multiplication problems. See <i>strategy</i> .
<b>repeating pattern</b>	A <i>pattern</i> generated by a core unit that repeats (e.g., A, B, B, A, B, B, A, B, B).
<b>represent</b>	To display one's <i>understanding</i> of a mathematical situation or concept by writing a mathematical <i>expression</i> or <i>equation</i> or by drawing a graph, table, or chart.
<b>rhombus</b>	A <i>quadrilateral</i> with four congruent sides.
<b>right angle</b>	An <i>angle</i> that <i>measures</i> 90 degrees.
<b>right triangle</b>	A triangle in which one angle measures 90 degrees.
<b>rotation</b>	A <i>transformation</i> that turns a <i>point</i> , <i>line</i> , or shape around a <i>point</i> .
<b>sample</b>	A <i>subset</i> of a <i>population</i> .
<b>scalene triangle</b>	A triangle with three sides of different <i>lengths</i> .
<b>scalar</b>	A quantity having magnitude but no direction, such as <i>mass</i> or <i>length</i> .
<b>scatter plot</b>	A graph of plotted <i>points</i> that depicts the relationship between two <i>variables</i> .
<b>scientific notation</b>	A way of expressing very large or very small numbers in the form $a \times 10^b$ , where <i>coefficient</i> $a$ is any real number and <i>exponent</i> $b$ is an <i>integer</i> .
<b>secant</b>	The <i>ratio</i> of the <i>length</i> of the <i>hypotenuse</i> to the length of the side lying adjacent to an <i>acute angle</i> in a right triangle.

<b>sector of a circle</b>	A portion of a <i>circle</i> formed by two <i>radii</i> and an arc.
<b>separate problems</b>	<i>Change problems</i> where the change amount is "taken away" from the initial amount. In these problems, the initial amount, change amount, or resulting amount is unknown.
<b>series</b>	The <i>sum</i> of the <i>terms</i> of a <i>sequence</i> .
<b>sequence</b>	An ordered list of numbers.
<b>set</b>	A group of objects or numbers that may or may not share a common <i>attribute</i> .
<b>set or discrete model (fractions)</b>	<i>Models</i> in which a <i>set</i> of individual objects is perceived as a whole and subsets make up <i>fractional</i> parts of the whole.
<b>shrinking pattern</b>	A <i>pattern</i> that diminishes in value or size from term to term (e.g., 10, 8, 6, 4).
<b>side</b>	A line segment joining two vertices or corners of a two-dimensional shape. See <i>vertex</i>
<b>similar figures (angles, shapes)</b>	Two figures are similar if their corresponding angles are <i>congruent angles</i> and corresponding sides are <i>proportional</i> .
<b>simple events</b>	An event with a single outcome (e.g., drawing the ace of spades from a standard deck of cards).
<b>simplest form fractions</b>	A <i>fraction</i> whose <i>numerator</i> and <i>denominator</i> share no <i>factors</i> other than the number one.
<b>sine</b>	The <i>ratio</i> of the <i>length</i> of the side lying opposite of an <i>acute angle</i> to the length of the <i>hypotenuse</i> in a right triangle.
<b>skip counting strategy</b>	To add, subtract, multiply, or divide using counting by twos, threes, fours, etc. See <i>strategy</i> .
<b>slope</b>	An <i>attribute</i> of a <i>line</i> describing its steepness and direction represented by a <i>ratio</i> of the increase in the <i>y</i> -coordinate to the increase in the <i>x</i> -coordinate.
<b>sort</b>	To select objects from a <i>set</i> to create one or more subsets of objects, where the objects in each subset share one or more common <i>attributes</i> .
<b>square</b>	A <i>quadrilateral</i> with four congruent sides and four <i>angles</i> measuring 90 degrees (i.e., <i>right angles</i> ).
<b>square matrix</b>	A <i>matrix</i> with the same number of rows and columns.

<b>square root</b>	The number whose square is a given number.
<b>square-root function</b>	A <i>function</i> where the outputs are the <i>square roots</i> of the inputs.
<b>standard deviation</b>	A <i>measure</i> of the spread of a data <i>set</i> .
<b>standard units of measurement</b>	Units of <i>measure</i> used commonly in a population or culture (e.g., U.S. customary units: inches, feet, ounces, pounds, cups, gallons; metric units: centimeters, meters, liters, kilograms).
<b>statistical inference</b>	The process of drawing conclusions about a <i>population</i> based on data collected from a <i>sample</i> .
<b>step function</b>	A <i>function</i> for which the <i>independent variable</i> is continuous and the <i>dependent variable</i> is discrete (e.g., postage costs, where the independent variable is the weight of a letter and the dependent variable is the number of stamps needed to mail the letter).
<b>straight angles</b>	An <i>angle</i> that <i>measures</i> 180 degrees.
<b>strategy</b>	A sequence of actions executed to make sense of a mathematical situation and/or obtain mathematical results.
<b>subitize</b>	To <i>recognize</i> directly the number of objects in a <i>set</i> without consciously using any mental or mathematical processes.
<b>subtraction strategies</b>	<i>Strategies</i> that help students to take away one number from another (e.g., forward counting, backward counting, <i>place value understanding</i> , <i>manipulatives</i> ).
<b>subtraction word problems</b>	A hypothetical situation expressed in words that requires students to find an unknown quantity using a subtraction operation or <i>strategy</i> .
<b>sum</b>	The result obtained by adding two quantities.
<b>supplementary angles</b>	Two <i>angles</i> whose <i>sum</i> is 180 degrees.
<b>surface area</b>	Total <i>area</i> of the surface of a three-dimensional object (e.g., the surface area of a cube with side measuring $b$ units is $6b^2$ ).
<b>symmetric function</b>	A symmetric <i>function</i> is unchanged by any <i>permutation</i> of its <i>variables</i> (e.g., the function's value is unchanged when one or more variables are exchanged with their <i>opposites</i> in the function).
<b>symmetric shape</b>	A shape that has one or more internal <i>lines</i> of symmetry, where the halves of the shape on either side of the line of symmetry are mirror images of each other.

<b>tally chart</b>	A table for recording frequency data, usually created by drawing strokes.
<b>tangent line</b>	A <i>line</i> that touches a curve at exactly one <i>point</i> (e.g., the tangent to a <i>circle</i> touches exactly one point on the <i>circumference</i> of the circle and is <i>perpendicular</i> to the <i>radius</i> at that point).
<b>term</b>	The numbers or <i>variables</i> in a <i>pattern</i> , <i>sequence</i> , <i>expression</i> , or <i>equation</i> separated by commas (pattern, sequence) or addition or subtraction symbols (expression, equation).
<b>theoretical probability</b>	The <i>ratio</i> of favorable outcomes to the total number of outcomes, based on logical analysis of the outcomes rather than experimental results.
<b>transformation</b>	Changes in the position, orientation, or size of an object, shape, or graph.
<b>translation</b>	A <i>transformation</i> that slides a <i>point</i> , <i>line</i> , shape, or graph to a different location.
<b>transversal</b>	A <i>line</i> that intersects a system of lines.
<b>triangle</b>	A polygon with three sides and three vertices.
<b>two-step word problems</b>	Word problems that require the use of two mathematical operations or steps to determine the solution.
<b>undefined slope</b>	The <i>slope</i> of a <i>vertical line</i> , where the amount of vertical change equals a non-zero real number, and the amount of <i>horizontal</i> change equals zero, yielding a slope <i>ratio</i> whose <i>denominator</i> equals zero.
<b>understand</b>	To <i>know</i> the conceptual and procedural meaning of a mathematical idea, connect this idea to related concepts and procedures, and apply it to learn new mathematical ideas.
<b>union of sets</b>	The <i>set</i> of all the elements from two or more sets.
<b>unit</b>	A group of countable objects that make up a whole.
<b>unit circle</b>	A <i>circle</i> with <i>radius</i> one centered at the <i>origin</i> .
<b>unit cube</b>	A cube with edge <i>lengths</i> of one unit and <i>volume</i> of one cubic unit.
<b>unit fraction</b>	A quantity formed by one part when a whole is <i>partitioned</i> into $n$ equal parts.
<b>unit rate</b>	The simplified value of a <i>ratio</i> whose <i>denominator</i> equals one (e.g., 60 miles per hour).
<b>unit square</b>	A <i>square</i> with edge <i>lengths</i> of one unit and <i>area</i> of one square unit.

<b>unitize</b>	The process of conceptualizing information in chunks or <i>units</i> (e.g., thinking 24 packs of cola as 4 six-packs, 2 dozens, 3 eight-packs).
<b>upper quartile</b>	The <i>median</i> of the upper half of a data <i>set</i> .
<b>variable</b>	A letter or a symbol that stands for an unknown quantity.
<b>vector</b>	A geometric quantity involving both magnitude and direction.
<b>vertex</b>	A <i>point</i> at which two <i>line segments</i> , <i>rays</i> , or <i>lines</i> meet to form an <i>angle</i> (e.g., the corners of a <i>polygon</i> ).
<b>vertex/corner</b>	A point where two sides of a polygon meet.
<b>vertical line</b>	A <i>line parallel</i> to the <i>y-axis</i> or at <i>right angles</i> to the <i>x-axis</i> .
<b>vertical angles</b>	Pairs of opposite <i>angles</i> formed by two <i>intersecting lines</i> , where the opposite angles are <i>congruent angles</i> and share a <i>vertex</i> where the lines intersect.
<b>vertical asymptote</b>	A <i>vertical line</i> corresponding to a value for which a <i>function</i> is undefined.
<b>vertical compression</b>	The squeezing of the graph or figure towards the <i>x-axis</i> .
<b>vertical stretching</b>	The stretching of the graph or figure away from the <i>x-axis</i> .
<b>volume</b>	The amount of space enclosed by a three-dimensional shape or an object.
<b>whole numbers</b>	The collection of natural numbers and zero (e.g., 0, 1, 2, 3, 4,.....).
<b>x-axis</b>	The <i>horizontal axis</i> on the <i>coordinate plane</i> .
<b>x-coordinate</b>	The first number in an ordered pair illustrating how many units the <i>point</i> is left or right of the <i>origin</i> on the <i>x-axis</i> (horizontal axis).
<b>x-intercept</b>	The <i>point</i> where a graph intersects the <i>x-axis</i> .
<b>y-axis</b>	The vertical axis on the <i>coordinate plane</i> .
<b>y-coordinate</b>	The second number in an ordered pair illustrating how many units the <i>point</i> is up or down from the <i>origin</i> on the <i>y-axis</i> (vertical axis).
<b>y-intercept</b>	The <i>point</i> where a graph intersects the <i>y-axis</i> .
<b>zero exponent property</b>	Any non-zero number raised to an <i>exponent</i> equaling zero equals one (i.e., $5^0 = 1$ ).

<b>zero matrix</b>	A <i>matrix</i> where each element is zero.
<b>zero property of division</b>	The <i>quotient</i> of zero divided by any non-zero number equals zero.
<b>zero property of multiplication</b>	The <i>product</i> of any real number and zero equals zero.
<b>zero slope</b>	The <i>slope</i> of <i>horizontal line</i> where the amount of vertical change equals zero and the amount of horizontal change equals a non-zero real number, yielding a slope <i>ratio</i> whose <i>numerator</i> equals zero.